Understand Love of Variety in Wireless Data Market Under Sponsored Data Plans

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***Abstract*—Sponsored Data Plan (SDP) is an emerging pricing model for the wireless data market where the Content Provider (CP) can sponsor the data usage for specific content on behalf of the users. This strategy sheds new light on the data pricing model and receives significant attention from the Internet Service Provider (ISP). However, the existing SDP studies consider traffic price (*e.g.*, sponsorship) as the only factor that affects user decision. The impact of other classic market features, such as the demand for a variety of contents (*i.e.*, love of variety), remains largely unclear. In this paper, we develop a new model to understand the love of variety in the wireless data market under SDPs. Our model has demonstrated that, such variety is important to understand the complex gaming between ISPs, CPs, and users in both short-run and long-run markets. For example, the analysis indicates that the advantage of CPs with higher revenue will be significantly reduced when users have a greater love of variety. Moreover, to help the ISP better adopt the proposed model in the real market, we also develop a practical method to calibrate the related parameters, which can also be applied to quantity the love of variety.**

***Index Terms*—Sponsored data plan (SDP), relative love of variety (RLV), competition among CPs.**

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I. Introduction

To HANDLE the explosive growth of mobile data usage, Sponsored Data Plans (SDPs) have surged as a promising and sustainable solution in recent years. This new type of smart data pricing enables collaboration between the Internet Service Provider (ISP) and the Content Provider (CP) by bridging their related revenues [2] from the content consumers (*i.e.*, users). As illustrated in Fig. 1, the CP can sponsor the data usage for specific content on behalf of the users from a given ISP. The users can, therefore, enjoy the content without impacting their postpaid or even prepaid data plan allowance. Meanwhile, unlike other solutions, which break the network neutrality to mitigate congestion, SDPs are compatible with network neutrality, allowing ISP to have sufficient funds to upgrade the infrastructure for mitigating congestion. Despite the elimination of mandatory net neutrality in some regions, *e.g.*, USA [3], [4], there are multiple countries that mandate network neutrality, *e.g.*, Canada, Brazil, India, in which SDP can enable CPs to subsidize their users without violating network neutrality.

To better understand this novel pricing strategy, early studies develop models to explore the interactions among users, ISPs and CPs. These studies show that SDPs create a triple-win situation [5]–[8]. To better adapt SDP in real marketing, recent studies develop models to analyze the impact of SDPs on the competition among different CPs [9], [10]. The conclusions of these models indicate that SDPs may benefit big CPs[1] who can afford a higher sponsored level, which brings serious concerns on SDPs. That is, SDP will lead to unfair competition in the market, which in turn will weaken the incentives for CPs to innovate. It is worth noting that the existing SDP studies only consider traffic price (*e.g.*, sponsorship) as the only factor that affects user decision. However, pricing is never the only factor in most real-world market models [1], [11]. Other important features, such as the demand for a variety of contents [1], [12], [13], should also be carefully considered in the SDP modeling.

Nowadays, people have become accustomed to a wide variety of services over the Internet, *e.g.*, e-shopping, online meetings, and online entertainments. This can be reflected in many practical applications and research areas (*e.g.*, app usage modeling [14], [15] and app usage prediction [16]). For

[1]In this work, a big CP is defined as one CP with higer per unit revenue. Therefore, the big CP can also be called more profitable CP, which has the opposite meaning to the small CP.

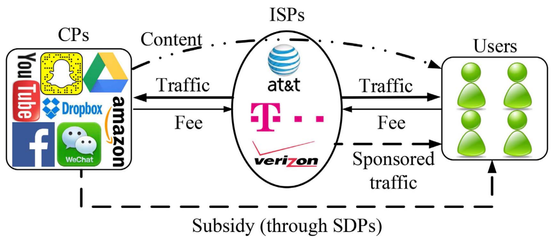


Fig. 1. The wireless data market under SDPs.

example, the average smartphone user has 80 apps on his or her phone, and uses 40 of them in a given month [17]. Another example, considering the frequent use of multiple services (*i.e.*, the intrinsic demand for various contents), recommendation system can use behavioral data from multiple sources to infer friendship with high-quality performance [18]. It is known that every user has a preference for such diversity [19]. Having a vast variety of products, services, and options is a value in itself. The demand for a variety of contents can be considered as a positive consumption externality, *i.e.*, the more contents are different, the more users feel satisfied. This fundamental feature is widely considered in most of the classic marketing models [12], [13]. Unfortunately, how this feature is going to affect the SDP system still remains largely unknown.

Modeling the detailed features of variety is challenging. In particular, the variety demand cannot simply be characterized as the user's cost. The nature of the variety demand is how hard it is to replace many different contents with a single content [20], [21]. In other words, if the variety demand is high, a user is more likely to access different contents during a fixed period. Moreover, it is also hard to directly measure users' variety demands and then integrate such a new index into the SDP market model. To address these problems, we characterize variety demand by using the notion of substitution and for the first time introduce the Relative Love of Variety (RLV) index into SDP. The key contributions we have made in this paper are summarized follows:

- With time being the benchmark, we characterize users' love of variety, and for the first time integrate RLV into an overall two-stage Stackelberg game model, analyzing the love of variety in the market under SDPs.
- Through extensive theoretical analysis and simulation experiments, the newly proposed model in this paper not only rectifies some conclusions of the past studies, but also derives some completely new results.
- To quantity the love of variety in the practical market, we develop a method to calibrate the RVL-related parameters through collectable data in the market.

Through analyzing the proposed model, we can explore RLV's potential impacts on SDPs, and the following summarizes the major findings and their implications:

- Different types of varieties (*i.e.*, variety-lovers, variety-avoiders and variety-free) may have different effects on multiple factors (*e.g.*, CP quantity and sponsored level), which can greatly affect the results of SDP analysis.
- CP sponsored level can be affected by CP/user quantity, which is a completely new result and has an impact on the choice of CP entering a certain market under SDPs.
- The influence of SDPs is not as significant as we believed before. For example, big CPs have advantages over small CPs. But such advantages will quickly decrease when the users have a greater love of variety (*e.g.*, variety-lovers).
- ISP's strategy produces some new conclusions. For example, when users are variety-avoiders, greater data cap[2] may lead to an increase in the sponsored level, and the competition among CPs will become tougher.

The remainder of this paper is organized as follows. In Section II, we review related studies. Section III introduces our new model. Section IV presents the competition among CPs with considering variety demand. Furthermore, the impact of ISP's strategy is analyzed in Section V. In Section VI, we provide a method to obtain parameters for the real-world market, smoothing the gap between modeling and deployment. Section VII concludes this paper.

II. Related Studies

Nowadays ISPs typically obtain the majority of their revenue from users. However, this one-side pricing model becomes unviable since users have limits on how much they are willing to pay while their demand for bandwidth keeps increasing. The newly proposed SDP is a two-sided model that encourages CPs to transfer part of their revenue to users so as to revamp the constrained traffic usage [5]. The proposal of SDP and its emerging in practice made it important to study how SDP will affect the market. Early works study the broad impact of SDP on the users, CPs and ISPs. It has been demonstrated that SDPs overall create a more balanced market and can vitalize network expansion [6], [9]. Njoroge *et al.* find that through CP-side pricing, ISPs could secure higher surplus and maintain higher investment levels [23]. Hande *et al.* find that subsidizing the user's connectivity costs by pricing CPs benefits both users and CPs [24], since CPs can gain more revenue, *e.g.*, from advertising, when users consume more contents. To better adapt to the future 5G wireless network, Sun *et al.* propose a joint optimization scheme, in which CPs have to consider sponsoring strategies in both cellular network and edge caching networks [25].

Recent studies emphasize on the impact of SDPs on the competition among different CPs [6], [7], [9], [10]. Through analyzing the competition between one big CP and one small CP, Zhang *et al.* observe that SDPs favor the big CP in the long-run competition, but favor the small CP in the short-run competition [9]. A two-class service model with consideration of Quality of Services (QoS) [10] shows that SDPs may increase the imbalance in revenue distribution among CPs. Through one model on regulated sponsorship competition among CPs, Ma *et al.* find that the main reason that certain CPs might be harmed is the high access prices [6]. Joe-Wong *et al.* [7] find that sponsorship favors less cost constrained CPs and more cost constrained users. These studies

[2]Each user has a total traffic usage limitation (or data cap [22]) by paying a fixed fee, which will be described in detail in Section III.

enable us to thoroughly understand the complicated interplay among stakeholders in the market under SDPs, especially the competition among CPs.

We find that all these models are limited in the sense that they only consider the price as the factor affecting the decisions of users. This may not be precise. In fact, consumers have an inherent demand for diverse of products, which plays a pivotal role in market decision-making. For example, to analyze monopolistic competition and product diversity, Dixit and Stiglitz [26] proposed the Dixit-Stiglitz model. In the utility function of this model, when the consumption level equals to zero, the marginal utility is infinite, indicating the characteristics of preference diversity. And the variant of this model has been used in numerous macroeconomic literatures. In Krugman's love of variety model of international trade [12], consumers always get positive effects regardless of consumer products, but the marginal utility is diminishing, which implicitly demonstrates the consumers' love of variety. Similar phenomena exist not only in the economic field but also in many other fields. Hamlen [13] finds that in the singles and albums markets, there are many non-quality factors affecting the market demand, which are also ascribed to the consumers' love of variety. Overall, the consumers' love of variety is widely found in various markets, but its impacts on the market under SDPs remain largely unclear. For a deeper understanding of the impacts, consumers' love of variety should also be considered in the economic models.

In this paper, we model variety as an intrinsic factor and reconsider the competition among CPs in the wireless data market under SDPs. Our new model rectifies certain conclusions of the past studies and derives some completely new results, which prove that love of variety in the market cannot be neglected. Overall, through our new model, various stakeholders (*i.e.*, ISP, CPs and users) in the wireless data market could have a more comprehensive understanding of the application of SDP in reality, which is conducive to promoting the further development of SDP.

III. General Model

To fully and concisely analyze the competition among CPs, we focus on the situation with the single ISP,[3] like [10], [28], [29], which is common in reality. Thus, the market consists of three stakeholders: a set of potential CPs $\mathcal{N}$, where $N = |\mathcal{N}|$ and a specific CP with index i, which is denoted by CP_i, a set of users $\mathcal{L}$, where $L = |\mathcal{L}|$, and a monopolistic ISP which provides the link capacity μ. We also denote $\tilde{\mathcal{N}}$ as the set of incumbent CPs, where $\tilde{\mathcal{N}} \subseteq \mathcal{N}$ and $\tilde{N} = |\tilde{\mathcal{N}}|$. Then we can denote the system as a quadruple $(\mathcal{N}, \tilde{\mathcal{N}}, \mu, L)$.

For the sake of clarity, Table I lists the major notions in market. It is worth noting that the specific meaning of the symbol also depends on its superscript and subscript. All the parameters except ρ, a, b, can obtain their real value from

[3]We set up this model not only for mathematical simplicity, but also capture one ISP's monopoly access power for a majority of CPs. Current long-term contracts also limit end users' transition from one ISP to another. In addition, if the multiple ISPs could form an unified coalition, our model can be extended into the market with multiple ISPs and our results will hold. Otherwise, the variety matters may different in oligopolistic market [27].

TABLE I
MAJOR NOTATION LIST

Sym.	Description
v_i	CP_i's revenue obtained from per unit content, *a.k.a.*, *per unit revenue* of CP_i
α_i	The average traffic consumption per unit time towards CP_i, *a.k.a.*, *rate of traffic consumption*
h_i	The sponsored traffic fraction provided by CP_i for its users, *a.k.a.*, *sponsored level*
$\bar{h}_i$	User's afforded traffic fraction in consuming the content of CP_i, *i.e.*, $1 - h_i$, *a.k.a.*, *afforded level*
$\hat{t}_i$	The time limitation of a user towards the content from CP_i
p	The unit price charged to CPs for the sponsored traffic, *a.k.a.*, *sponsored price*
q	The unit price charged to CPs for the connection service
H	The total traffic usage limitation in a data plan, which is decided by ISP and also called data cap
μ	The link capacity provided by the ISP
ρ	The indicator of user's preference for variety

TABLE II
FREQUENTLY USED TERMS

Term	Description
RLV (*i.e.*, r_u)	User's relative love of variety
Variety-lover (*i.e.*, i-RLV)	User prefers a greater variety against an increase in the content access time
Variety-avoider (*i.e.*, d-RLV)	User prefers a smaller variety against an increase in the content access time
Variety-free (*i.e.*, c-RLV)	User variety demand is independent with the content access time
Big CPs	CPs with higher per unit revenue

CP and ISP. Especially the parameter ρ, *a.k.a.*, the indicator of user's preference for variety, can hardly obtain from the market. We can only obtain its approximation by other ways, such as the method proposed in Section VI. In addition, we list the frequently used terms in Table II for a more concise understanding of the subsequent content.

In this section, we first model the behaviors of end users. And we introduce how to use RLV to capture their variety demand. We then model the utilities and behaviors of the CPs and the ISP, respectively. Finally, we model the overall market as a two-stage Stackelberg game.

A. Behaviors of End Users

To portray the behaviors of end users as realistically as possible, while integrating user's love of variety into the newly proposed model, we use a *time* vector $\mathbf{t} = t_{i\in\mathcal{N}}$ to represent the user's consumption in contents of different CPs. Here t_i indicates user's access time in a certain CP_i during a fixed period, *e.g.*, one month. We note that rather than the traffic volume, the utility of end users depends on the

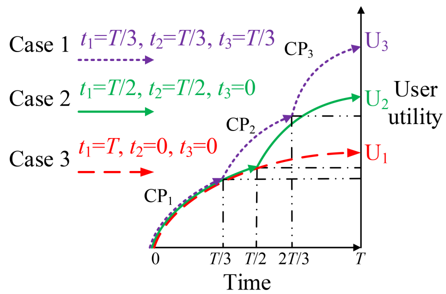


Fig. 2. An illustration for RLV. Given the same amount of total time T, user utility will increase if he or she consumes more different contents, since the marginal utility u' for single content always decreases with more time consumed.

access time, *i.e.*, t_i. We thus define the user's utility function as $u(t_i)$. We assume that $u(0) = 0$ and $u(t_i)$ is a strictly increasing and concave function, which is consistent with consumer utility characteristics in the Krugman's model [12]. Intuitively, a longer time means a higher user utility, but a smaller marginal user utility. We assume that the utilities from different CPs are additive. Then the aggregated utility of a user is $\sum_{i\in\mathcal{N}} u(t_i)$.

1) Variety/RLV: We now introduce the *variety* used in this paper. Intrinsically, one challenge is to quantify the willingness of a user to exchange one content x to another content y. We address the exchange of two contents by using the concept of *elasticity* [30]. Another challenge is that each user consumes different contents and we need to quantify the willingness of exchanging multiple contents. We address this issue by using a benchmark: each content is first exchanged to this benchmark. In this paper, we select *time* as the benchmark. Since time is equally valuable to anyone with any CP in terms of the abstract concept of willingness, it has the ability to be viewed as any content. This is also one of the reasons why we choose time to define the user's utility function. We first present the formal definition of elasticity.

Definition 1 (Elasticity): For two variables x and y, the x-elasticity of y is define as $\epsilon_x^y = -\frac{\partial y}{\partial x}\frac{x}{y}$.

The elasticity can be interpreted as the percentage change in y in response to the percentage change in x. The larger elasticity implies y is more sensitive to variation of x. To depict user's preferences for a variety of contents, we define RLV through elasticity.

Definition 2 (Relative Love of Variety (RLV)): The user's RLV is the elasticity of the marginal utility with respect to the consumption level t_i,

$$r_u(t_i) = \epsilon_{t_i}^{u'} = -\frac{u'' t_i}{u'} > 0. \tag{1}$$

As formerly notified, we use time as a media to make different contents exchangeable. And we show an illustration to understand RLV in Fig. 2. From Definition 2, we can see that the value of RLV implies whether users are willing to exchange their access time for higher marginal utility. That is, RLV can reflect the substitutability of the content. Note that the definition of RLV is with respect to a particular CP_i. It can be used to compare the substitutability of different CPs relative to the specific user. In addition, when we want to analyze the overall RLV difference between different users, for each user, we can use the average (or sum) of the RVLs from all CPs in the market to describe the overall RLV level.

2) Behavior of Users: We define the *rate of traffic consumption* as user's average traffic consumption per unit time towards CP_i, which is usually less than the bandwidth requirement. Different CPs may have different rates of traffic consumption. For example, a user may watch movies on YouTube and do shopping on Amazon for the same time duration, but the traffic he or she consumed on watching movies is obviously greater than that on shopping. Let α_i be the *rate of traffic consumption* for CP_i. Then, the traffic volume a user consumes on CP_i is $\alpha_i t_i$. With the SDP, the traffic volume can be partially sponsored. Let $h_i \in [0, 1]$ be the sponsored traffic fraction provided by CP_i for a user in consuming its content (we call it *sponsored level* hereafter). Let $\bar{h}_i = 1 - h_i$ be user's afforded traffic fraction in consuming the content of CP_i (we call it *afforded level* hereafter). Then, the traffic volume that a user needs to pay for is $\sum_{i\in\mathcal{N}} \bar{h}_i \alpha_i t_i$, which will be accumulated in user's cap quota. Under the present tiered pricing scheme provided by the ISP, each user has a total traffic usage limitation (or data cap [22]) by paying a fixed fee, which is denoted by H, *e.g.*, $H = 10$GB per month. The additional usage beyond the cap will be charged by a much higher price. As Zheng *et al.* [31], [32] have proved, users with such data cap have a strong incentive to plan their usage per month. That is, in reality, users do usually limit their usage below this cap due to the high fee charged for beyond. Thus, it is reasonable to assume that rational user's usage is below the cap. Under the assumption, user's access fee is a constant and does not affect any result. Therefore, we omit user's access fee in his or her utility formula. As mentioned in Section I, for each user, the monthly average number of apps used is up to 40 (*i.e.*, 40 CPs) [17]. It is impossible to use total usage time on one CP. Even if there are similar cases, they are very few exceptions. Therefore, each user also has time limitation on different CPs, *e.g.*, 18% of total usage time is spent on music [33]. The time limitation for CP_i is denoted by $\hat{t}_i$. Taking these constraints into account, an user $l \in \mathcal{L}$ can maximize his or her utility as follows,

$$\begin{aligned} &\max_{\mathbf{t}} \ \mathcal{U}_l = \sum_{i=1}^{N} u(t_i), \\ &s.t. \ \sum_{i=1}^{N} \bar{h}_i \alpha_i t_i \le H, \quad t_i \in [0, \hat{t}_i]. \end{aligned} \tag{2}$$

The above optimization can be solved by the Lagrange Multiplier with the optimal solution as follows.

Lemma 1: The optimal access time of a user towards the content from CP_i, denoted as t_i^*,

$$t_i^* = \min\{\hat{t}_i, u'^{-1}(\lambda \bar{h}_i \alpha_i)\}, \tag{3}$$

where λ is the Lagrange multiplier associated with the cap constraint.

Proof of Lemma 1: We introduce the Lagrange multiplier λ for the cap constraint, then we have the KKT condition as

$\sum_{i=1}^{N} u'(t_i) = \lambda(\sum_{i=1}^{N} \bar{h}_i\alpha_i)$. Then $u'(t_i) = \lambda\bar{h}_i\alpha_i$. Note that $t_i \leq \hat{t}_i$, thus $t_i^* = \min\{\hat{t}_i, u'^{-1}(\lambda\bar{h}_i\alpha_i)\}$. ■

We now study the relationship between RLV and the sponsored level. We first give a definition on Sponsoring-Response Elasticity (SRE) and then link RLV and SRE by Lemma 2.

Definition 3 (Sponsoring-Response Elasticity (SRE)): The SRE of a user is the elasticity of time t_i with respect to afforded level $\bar{h}_i$ after sponsoring, i.e., $\epsilon_{\bar{h}_i}^{t_i}$.

Lemma 2: SRE is equal to the inverse of RLV, i.e.,

$$\epsilon_{\bar{h}_i}^{t_i^*} = -\frac{\bar{h}_i}{t_i^*}\frac{\partial t_i^*}{\partial \bar{h}_i} = \frac{1}{r_u(t_i^*)}, \quad t_i^* \in (0, \hat{t}_i). \tag{4}$$

Proof of Lemma 2: From Lemma 1, $\bar{h}_i = \frac{u'(t_i^*)}{\lambda\alpha_i}$, which follows $\frac{\partial t_i^*}{\partial \bar{h}_i} = \frac{\lambda\alpha_i}{u''(t_i^*)}$. Thus, $\epsilon_{\bar{h}_i}^{t_i^*} = -\frac{\bar{h}_i}{t_i^*}\frac{\partial t_i^*}{\partial \bar{h}_i} = -\frac{u'(t_i^*)}{t_i^* u''(t_i^*)}$. According to Definition 2, we have $\epsilon_{\bar{h}_i}^{t_i^*} = \frac{1}{r_u(t_i^*)}$. ■

From the proof, we can see that both RLV and SRE are linked to the user utility function $u'(t_i)$. This lemma is important because RLV, *i.e.*, a separately defined index, can now be integrated into the model and optimization through $\bar{h}_i$.

B. RLV Classification and Examples

As mentioned above, *time* is chosen as the benchmark to solve the challenge of quantifying the willingness of exchanging multiple contents. Therefore, time-based user utility is closely related to RLV. In fact, not all users like to exchange among multiple. For example, Silva *et al.* [34] has demonstrated the transition diversity among users, which can indirectly reflect whether the user likes to use a variety of different contents. It demonstrates that 18% of users will only exchange between one or two applications, while most users prefer to exchange between various applications. Accordingly, we further classify users into three categories: *variety-lover*, *variety-avoider* and *variety-free*, to denote RLV increases with t_i (i-RLV, *i.e.*, $r_u'(t_i) > 0$), RLV decreases with t_i (d-RLV, *i.e.*, $r_u'(t_i) < 0$), and RLV is constant with t_i (c-RLV, *i.e.*, $r_u'(t_i) = 0$), respectively. We emphasize again that all users have diversity preferences. The difference among these three categories is that for variety-lovers the RLV will increase when t_i increases, and variety-avoiders act oppositely. In addition, the RLV for variety-free category is independent of t_i.

To depict user's preference of different types of RLV, a general utility function can be defined as follows,

$$u(t_i) = \frac{1}{1-\rho}[(a+t_i)^{1-\rho} - a^{1-\rho}] + bt_i, \tag{5}$$

where $a \geq 0$, $b \geq 0$ and $0 < \rho < 1$.

Note that the user might have different utilities for different CPs with the same time usage. For the sake of simplicity, we adopt the same utility function for different CPs. It is conducive to analyzing the love of variety, and cannot conflict with the results of specifying different utility functions for different CPs. Here we call ρ as the RLV index. With the different value of parameters a and b, the utility function indicates a certain RLV type of users. For example, when $a = 1$, $b = 0$, the corresponding RLV $r_u(t_i) = \frac{\rho}{1+1/t_i}$ increases with t_i (i-RLV). When $a = 0$, $b = 1$, the corresponding RLV $r_u(t_i) = \frac{\rho}{1+t_i^{\rho}}$ decreases with t_i (d-RLV).

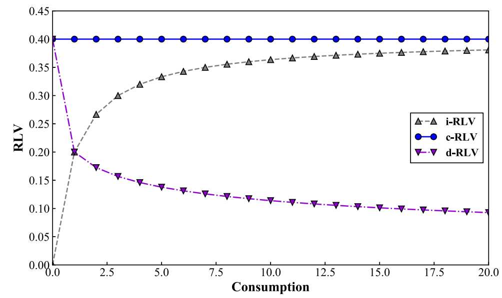


Fig. 3. The three types of RLV, including i-RLV, c-RLV and d-RLV.

When $a = 0$ and $b = 0$, the corresponding RLV is a constant ρ (c-RLV). Moreover, we plot the curves to illustrate these three types of RLV in Fig. 3, in which the RLV index ρ is set as 0.4.

C. Behaviors of CPs and the ISP

1) Utility and Behaviors of CPs: The CP_i's revenue obtained from per unit content (we call it *per unit revenue* hereafter) is denoted by v_i. It is well known that CPs may have different per unit revenue, such as Google and YouTube. Although all CPs can sponsor traffic volumes for their users so that more users access more contents, there may be differences in *sponsored level* (*i.e.*, h_i). In other words, h_i differs from different CPs. For example, $h_i = 0.5$ means that the relevant expenses are shared equally between the CP and the user. In particular, when $h_i = 0$, it means that CP_i gives up the opportunity to participate in the sponsored data plan. The cost of CP_i consists of three parts: *(i)* the cost $q \geq 0$ for the connection service of per unit traffic; *(ii)* the additional cost $p \geq 0$ for the per unit fee an ISP charges the CPs for the sponsored traffic (we call it *sponsored price* hereafter); *(iii)* the cost of entering to the market s_i. For homogeneous users, the total traffic usage for CP_i is $L\alpha_i t_i$, where L refers to the user quantity in the market. In fact, our model is also appropriate for heterogeneous users whose traffic usage is different for different CPs. Here we only consider homogeneous users for mathematical simplicity. Let ϕ_i be the utility function of CP_i, then the decision of CP_i is to choose appropriate h_i to maximize ϕ_i, formally,

$$\max_{h_i \in [0,1]} \quad \phi_i = (v_i - ph_i - q)L\alpha_i t_i - s_i. \tag{6}$$

2) Utility and Behaviors of the ISP: The revenue of the ISP mainly comes from two sources: the unit price charged to CPs for the connection service (*i.e.*, q), and the sponsored price charged to CPs for the sponsored traffic (*i.e.*, p). Note that we treat the connection service price and the sponsored price for different CPs as equal so as to avoid the arguing about network neutrality rules. We omit the price charged to end users because it is only a constant under the cap scheme. Let the traffic volume transmitted between CPs and users be η and $\eta = \sum_{i\in\mathcal{N}} L\alpha_i t_i$. When the traffic demand (*i.e.*, η) exceeds the capacity (*i.e.*, μ), the system falls into congestion which generates operating costs to ISP. We define the congestion cost as a function $c(\eta, \mu)$, which is convex and monotone increasing in η. In practice, the higher congestion implies worse QoS, thus users may decrease their usage or even transfer to other ISPs, which will reduce the ISP's profit [30]. Then the ISP will consider the negative effects brought by congestion when ISP make decisions. Therefore, we adopt the cost function to depict such profit reduction. Let π be the utility function of the ISP, then the decision of ISP is to choose appropriate p, q to maximize π, formally,

$$\max_{\{p,q\}} \quad \pi = \sum_{i=1}^{N} (ph_i + q)L\alpha_i t_i - c(\eta, \mu). \tag{7}$$

One choice of $c(\eta, \mu)$ is the capacity sharing congestion function [35]. Let load rate ω be the ratio of the traffic demand over capacity, *i.e.*, $\omega = \eta/\mu$. A higher load rate means a higher level of network congestion. Then the congestion cost is defined as $c(\eta, \mu) = \chi\omega^{\delta}$, where χ is a congestion level fee to the ISP and $\delta \geq 1$ represents the load sensitivity. Clearly, $c(\eta, \mu)$ is continuous, increasing in η, decreasing in μ and $c(0, \mu) = 0$, $\lim_{\mu\to\infty} c(\eta, \mu) = 0$. We assume $c(\eta, \mu)$ is a twice differentiable and convex function with respect to ω.

D. A Two-Stage Stackelberg Game Model of the Market

To model the interaction between various stakeholders (*i.e.*, the ISP, CPs and users), the wireless data market illustrated in Fig. 1 has been modeled as a two-stage Stackelberg game, which consists of two stages. In the first stage, the monopolistic ISP is the first mover and CPs are the followers. The ISP decides the sponsored price for CPs, and the data cap for end users, *i.e.*, its strategy profile is $\mathcal{S}^I \in \{(p, H)\}$. In the second stage, the CPs form a simultaneous game themselves. Each CP_i decides the sponsored level for end users, *i.e.*, its strategy profile is $\mathcal{S}_i^P \in \{h_i\}$. The outcome is determined by backward induction. In the second stage, $\mathcal{S}^I$ is considered to be fixed. Each CP_i decides its optimal sponsoring strategy. Then, in the first stage, the ISP decides its optimal price and data cap based on the outcome of the CPs decisions.

It is worth noting that many existing studies [10], [36] have adopted similar mathematical models (*i.e.*, Stackelberg games), which have the ability to describe the decision hierarchy in the wireless data markets. This is mainly owning to that the market structure has price2respond property. Note that we do not include the decision of q into the ISP's strategy profile. This is because we want to focus on the sponsored data scheme provided by CPs, which influences end user's decisions, but has limited impacts on q. Therefore, we assume q is predetermined and known. More precisely, we emphasize on the competition among CPs, *i.e.*, the simultaneous game in the second stage of the game, which is analyzed in Section IV. And we analyze the ISP decisions and the impact of ISP decisions on the competition among CPs in Section V.

IV. Competition Among CPs

Different from the previous mechanism of traffic pricing by the ISP alone, SDPs introduce CPs into pricing market for the first time, which has indirectly affected the pricing process of the overall market by adjusting the sponsored level. This inevitably triggers competition among different CPs. To comprehensively analyze this novel competitive approach, we focus on the CPs behaviors in two different scenarios.

We first study the market with homogeneous CPs, *i.e.*, the CPs with the same rate of traffic consumption α and the same per unit revenue v. We also assume that they have the same cost of entering the market, denoted by s. For example, we can consider the CPs that provide video services to be homogeneous since they have the same rate of traffic consumption. Note that these CPs can provide different contents, thus the market has variety. This scenario is useful since CPs with video services are heavily affected by this new SDP pricing model and they are mostly eager to understand the impact of SDPs on their competition. If we consider that only CPs with video services conduct sponsorship, then it is a market with homogeneous CPs. In addition, we study the market with heterogeneous CPs, which is a general and comprehensive case.

To reveal more insights into the competition among different CPs, we further analyze both the short-run and the long-run equilibrium states for each market (*i.e.*, the market with homogeneous CPs and the market with heterogeneous CPs). In the short-run equilibrium, the number of CPs is fixed and no CP in the market finds it profitable to change its sponsored level unilaterally. Conversely, in the long-run equilibrium, CPs can enter and exit freely, till no new CP wants to join or existing CPs want to leave.

We now first analyze the optimal decision of CPs, and then analyze the equilibrium state of the simultaneous game of the CPs. These help our analysis in Subsection IV-A and Subsection IV-B on the detailed CP behaviors.

From Equation (3), we have $t_i^* = u'^{-1}(\lambda\bar{h}_i\alpha_i)$ for any $t_i^* \in (0, \hat{t}_i)$. We can see that user's optimal time varies with sponsored level. For the mathematical simplicity, we treat the t_i as t_i^* hereafter. Thus, we have

$$\bar{h}_i = \frac{u'(t_i)}{\lambda\alpha_i}. \tag{8}$$

With Equation (8), the optimization problem of CP_i, *i.e.*, Equation (6), is rewritten by

$$\max_{t_i\in(0,\hat{t}_i)} \phi_i = (\frac{u'(t_i)}{\lambda} - \alpha_i A_i)pLt_i - s_i, \tag{9}$$

where $A_i = \frac{p+q-v_i}{p}$. Here, we abuse the notation a little and let $z_i = \alpha_i A_i$ be the cost of CP_i (we also call it CP's *type*). If $z_i > 0$, CP_i has a positive cost. A higher (lower) cost usually indicates higher (lower) α_i and smaller (higher) v_i, which demonstrates CP_i has a smaller (higher) advantage in the market competition. If $z_i < 0$, CP_i has a negative cost, *i.e.*, it always benefits from more traffic usage.

Note that when making decisions on its optimal sponsored level, a CP may influence the Lagrange multiplier λ and the traffic consumption of other CPs. Nevertheless, we consider the case where the number of CPs is large and such influence is ignorable. For example, there were about 2.2 million apps available to download in Apple APP Store and users had an average of 88.7 apps installed on their smartphones [37]–[39].

Thus, we assume that CPs are price takers who are not influential enough to affect the market price, like [40]–[42]. Under the assumption, each CP accurately treats the Lagrange multiplier λ as an exogenous parameter and estimates the equilibrium value of λ. Having done this, the CP behaves like a monopolist on its market and thus maximizes its profit.

Let $D_i \equiv \frac{\partial u(t_i)}{\partial t_i}$, $D_i' \equiv \frac{\partial D_i}{\partial t_i}$. The first-order condition of ϕ_i respects to t_i can be written as

$$D_i + t_i D_i' = [1 - r_u(t_i)]D_i = \lambda \alpha_i A_i. \tag{10}$$

Recall that we have assumed that the user utility function is strictly concave, which implies that $D > 0$ and $D' < 0$. It is thus sufficient to assume that the following Inada conditions [43] hold as follows,

$$\lim_{t_i \to 0} D_i = \infty, \quad \lim_{t_i \to \infty} D_i = 0. \tag{11}$$

When $\lambda \alpha_i A_i > 0$, we have

$$0 < r_u(t_i) < 1, \text{ for any } t_i. \tag{12}$$

The conditions (11) and (12) imply that

$$\lim_{t_i \to 0} (1 - r_u(t_i))D_i = \infty, \quad \lim_{t_i \to \infty} (1 - r_u(t_i))D_i = 0. \tag{13}$$

The intermediate value theorem implies that Equation (10) has at least one positive solution.[4] When $\lambda \alpha_i A_i < 0$, the optimal time for CP_i approaches the maximum time $\hat{t}_i$. Furthermore, if the user utility function is strictly concave, Equation (10) has a unique solution and this solution can maximize the CP's profit. The uniqueness condition of the solution is equivalent to

$$r_{u'}(t_i) = -t_i \frac{D_i''}{D_i'} < 2. \tag{14}$$

In summary, we have the following lemma.

Lemma 3: If the conditions (11) (12) and (14) are satisfied, then for any $\lambda > 0$, there exists a unique optimal decision in equilibrium for CP_i in Equation (9), given by

$$h_i = 1 - \frac{u'(t_i)}{\lambda \alpha_i}, \; t_i = \min\{u'^{-1}(\frac{\lambda \alpha_i A_i}{1 - r_u(t_i)}), \hat{t}_i\}. \tag{15}$$

Proof of Lemma 3: Considering CP_i's utility ϕ_i, the first-order condition respects to t_i is $-pt\bar{h}' = v - p - q + p\bar{h}$, then we have $\bar{h} = \frac{p+q-v}{p\cdot(1-r_u(t_i))} = \frac{A}{1-r_u(t_i)}$. Substituting $\bar{h}$ in Equation (3), then we obtain the result. ■

Lemma 3 shows the sufficient conditions for the uniqueness of each CP's optimal decision. In fact, the condition (14) itself can guarantee such uniqueness. The conditions (11) and (12) guarantee that the optimal decision is reasonable and meaningful. For example, if $r_u(t_i) > 1$ for all $t_i \geq 0$, then for any $A_i > 0$ (even for $v_i > q$), $t_i = 0$. In other words, this means that CP_i can achieve its maximal profit when no user accesses its content. Clearly, this contradicts to the common sense. Note that the optimal decision here may not be in the equilibrium unless λ is the equilibrium value. In the next subsections, we will analyze the optimal decisions of CPs in the equilibrium state.

[4] In reality, the condition of $\lim_{t_i \to 0} D_i = \infty$ in Equation (11) may not hold. However, we make the setting to ensure the Equation (10) has a valid solution.

A. Homogeneous Content Providers

We now focus on the CPs behaviors in the first scenario, in which homogeneous CPs have the same features of α and v. Note that the same α and v do not imply that the CPs provide identical contents.

1) The Short-Run Equilibrium: In the short-run market, the quantity of incumbent CPs is fixed, that is, $\tilde{N}$ is a constant. We first study the optimal decision of the CPs in the equilibrium state. We then analyze the impact of CP quantity on the short-run equilibrium under the variety preference.

We have known Equation (10) has a single solution t_i. Note that all CPs are homogeneous and face with the same λ, so t_i and h_i are symmetric in equilibrium for all $i \in \tilde{\mathcal{N}}$. Let t and $\bar{h} = 1 - h$ be the symmetric results for end users and CPs. In the equilibrium, if the time maximum is not reached, the cap should be fully filled, *i.e.*,

$$t = \frac{H}{\tilde{N}\alpha\bar{h}}. \tag{16}$$

With this condition and Lemma 3, we can estimate the λ in the equilibrium and thus get the optimal solution. More specifically, we have the following proposition.

Proposition 1: In the market with homogeneous CPs, the optimal solution in the equilibrium is

$$\bar{h} = \max\left\{\frac{A}{1 - r_u(\frac{H}{\tilde{N}\alpha\bar{h}})}, \frac{H}{\tilde{N}\alpha\hat{t}}\right\}, \tag{17}$$

where $t = \min\left\{\frac{H}{\tilde{N}\alpha\bar{h}}, \hat{t}\right\}$.

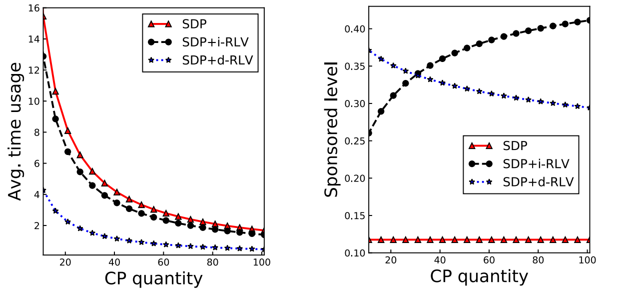
Proof of Proposition 1: From CP's utility Equation (6), the first-order condition respects to t is $-pt\bar{h}' = v - p - q + p\bar{h}$ $\Rightarrow r_u(t) = \frac{v-p-q+p\bar{h}}{p\bar{h}} \Rightarrow \bar{h} = \frac{p+q-v}{p\cdot(1-r_u(t))}$. According to Equation (16), thus $\bar{h} = \frac{A}{1-r_u(H/\tilde{N}\alpha\bar{h})}$. When $t = \hat{t}$, then $\bar{h} = \frac{H}{\tilde{N}\alpha\hat{t}}$, thus we obtain the result. ■

This proposition captures the characteristic of CP's optimal solution in the equilibrium. If $t < \hat{t}$, the RLV affects the optimal sponsored level. This condition is satisfied if and only if $\bar{h} > \frac{H}{\tilde{N}\alpha\hat{t}}$. If $t = \hat{t}$, the sponsored level also approaches its maximum, *i.e.*, $1 - \frac{H}{\tilde{N}\alpha\hat{t}}$.

Theorem 1 (CP Quantity Effect): In the short-run equilibrium, if $t < \hat{t}$, then the sponsored level is higher (lower) in the market with the larger quantity of incumbent CPs when $r_u' > 0$ ($r_u' < 0$). Otherwise, the sponsored level is always proportional to the quantity of incumbent CPs.

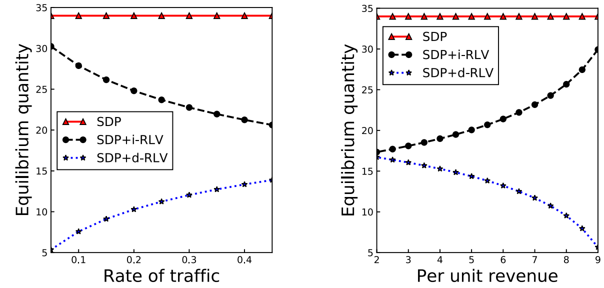
Proof of Theorem 1: When $t < \hat{t}$, we differentiate the $\bar{h}$ with respect to $\tilde{N}$ from the Proposition 1, and we have the below expression $(1 - r_u + tr_u')\frac{\tilde{N}}{\bar{h}}\frac{d\bar{h}}{d\tilde{N}} = -tr_u'$. If $r_u' > 0$, then $\bar{h}$ decreases with $\tilde{N}$, that is, the sponsored proportion increases with $\tilde{N}$, and vice versa. When $t = \hat{t}$, $\bar{h} = \frac{H}{\tilde{N}\alpha\hat{t}}$, then $\frac{d\bar{h}}{d\tilde{N}} = -\frac{H}{\tilde{N}^2\alpha\hat{t}} < 0$, that is, h increases with $\tilde{N}$. ■

When users are variety-lovers, *i.e.*, $r_u' > 0$, the larger $\tilde{N}$ in the market means smaller consumption level and thus smaller RLV. The variety of contents are better substituted with each other and the competition is more intense. Under this circumstance, the CPs have to increase the sponsored level. On the contrary, when users are variety-avoiders, *i.e.*, $r_u' < 0$, the larger $\tilde{N}$ leads to a higher RLV. The contents



(a) Time usage vs. $\tilde{N}$ (b) Sponsored level vs. $\tilde{N}$

Fig. 4. Impact of $\tilde{N}$ on the short-run market.



(a) CP quantity vs. α (b) CP quantity vs. v

Fig. 5. Impacts of α and v on equilibrium CP quantity in the long-run market.

become more differentiated. This time, the competition in the market is relatively moderate, which leads to CPs decreasing the sponsored level.

To further clarify the theorem, we perform some simulation experiments, which are independent of the content of Theorem 1. As illustrated in Fig. 4(a), we can see that no matter what type the RLV is, the average time usage always decreases as the CP quantity increases. This finding is consistent with previous studies. Fig. 4(b) demonstrates the impact of CP quantity on sponsored level. Moreover, these impacts are different for different RLV category markets, which is consistent with Theorem 1.

2) The Long-Run Equilibrium: In the long-run market, CPs could enter or exit according to their operating profit, that is, $\tilde{N}$ could change. We first analyze the quantity of CPs in the equilibrium state. Then we study the comparison of the market with and without SDPs in the consideration with RLV.

When a potential CP can earn positive profit, it will enter the market, which reduces the revenue of incumbent CPs. In the equilibrium, no CP has the incentive to enter the market, *i.e.*, all CPs in the market earn zero profit. More formally,

$$(\bar{h} - A)pL\alpha t = s. \tag{18}$$

With Equation (18) and Proposition 1, we can capture the equilibrium number of CPs in the long-run market by the following proposition.

Proposition 2: In the market with homogeneous CPs, the number of CPs in the long-run equilibrium satisfies

$$\tilde{N}^* = \min\left\{\frac{pHLM}{s}, \frac{H}{\alpha\hat{t}A + \bar{s}/p}\right\}, \tag{19}$$

where $M = r_u\left[\frac{s}{L\alpha p}\frac{1}{A}\left(\frac{1}{M} - 1\right)\right]$, *and* $\bar{s} = \frac{s}{L}$.

Proof of Proposition 2: According to Lemma 2 and Equation (8), we have $M = \frac{\bar{h}-A}{\bar{h}} = 1/\epsilon_{\bar{h}_i}^{t_i} = r_u(t)$.

From Equation (18), we have $p \cdot (\bar{h} - A)t = \frac{s}{L\alpha} \rightarrow pA \cdot \frac{M}{1-M}t = \frac{s}{L\alpha} \rightarrow t = \frac{s}{\alpha p}\frac{1}{LA}(\frac{1}{M} - 1)$, so $M = r_u(\frac{s}{\alpha p}\frac{1}{LA}(\frac{1}{M} - 1))$.

From the constraint condition in Equation (2), then we have $\tilde{N}\bar{h}\alpha_i t_i = H$, thus $\alpha_i t_i = \frac{H}{\tilde{N}\bar{h}}$. According to Equation (18), then $\frac{\bar{h}-A}{\bar{h}}pLH = \tilde{N}s$, thus $\tilde{N} = \frac{pHLM}{s}$.

When $t = \hat{t}$, $\tilde{N}$ can achieve the maximum value, denoted by $\bar{N}$. From Proposition 1, we can have $\bar{h} = \frac{H}{\bar{N}\alpha\hat{t}}$. According to Equation (18), $\bar{h} - A = \frac{s}{L\alpha\hat{t}\cdot p} \rightarrow \frac{H}{\bar{N}\alpha\hat{t}} = \frac{s}{L\alpha\hat{t}} + A = \frac{s+A\cdot L\alpha\hat{t}\cdot p}{L\alpha\hat{t}\cdot p} \rightarrow \frac{H}{\bar{N}} = \frac{s+AL\cdot\alpha\hat{t}\cdot p}{L\cdot p} \rightarrow \bar{N} = \frac{HLp}{s+AL\cdot\alpha\hat{t}\cdot p} = \frac{H}{\frac{s}{Lp}+\alpha\hat{t}A} = \frac{H}{\frac{\bar{s}}{p}+\alpha\hat{t}A}$, where $\bar{s} = \frac{s}{L}$. Thus, we obtain the results. ■

There are two cases in the equilibrium. When $t < \hat{t}$, the RLV affects the number of CPs in the equilibrium. In particular, if the RLV is a constant, then the number of CPs is independent with the characteristics of CPs, *e.g.*, α and v, unless $t = \hat{t}$. In fact, $\tilde{N}^*$ represents the optimal CP capacity for a given market. To further clarify the relationship between the characteristics of CPs and the equilibrium CP quantity, Fig. 5 illustrates some simulation results. It can be found from Fig. 5(a) that, when users are variety-lovers (*i.e.*, $r' > 0$), homogeneous CPs with a better technology level (*i.e.*, smaller α) can facilitate the expansion of CP capacity. And when users are variety-avoiders (*i.e.*, $r' < 0$), the situation is the opposite, which proves the different impact of RLV on the market. Regarding another characteristic (*i.e.*, v in Fig. 5(b)), when $r' > 0$ ($r' < 0$), CPs with a larger (smaller) per unit revenue can be more conducive to the expansion of the market. When $t = \hat{t}$, if some new CP enters the market, the sponsored level will become higher according to Theorem 1. This reduces all CPs' revenue. And the negative profit prevents this new CP entering the market.

Next, we assume that CPs do not participate in the SDP. And the equilibrium quantity of these CPs is denoted as $\tilde{N}^{no}$, then we have the following theorem.

Theorem 2 (Market Variety): In the long-run market, if $A \geq 0$, *then* $\frac{\tilde{N}^{no}}{\tilde{N}^*} > 1$ *and decreases with RLV. Otherwise, the relationship is reversed.*

Proof of Theorem 2: We have defined $M \equiv \frac{\bar{h}-A}{\bar{h}}$, then $\bar{h} = \frac{A}{1-M}$. When $h = 0$ and $\bar{h} = 1$, then $A = 1$-M $= \frac{p+q-v}{p}$, $M = 1 - \frac{p+q-v}{p} = \frac{v-q}{p}$. From Equation (19), $\tilde{N}^* = \frac{pHLM}{s}$ and $\tilde{N}^{no} = \frac{pHL}{s}\frac{v-q}{p}$, thus $\tilde{N}^{no}/\tilde{N}^* = \frac{(v-q)/p}{M} = \frac{(v-q)(1-h)}{v-ph-q}$. When $A \geq 0$, $(v-q)(1-h) - (v-ph-q) = (p+q-v)h > 0 \Rightarrow (v-q)(1-h) > (v-ph-q)$. Thus $\frac{\tilde{N}^{no}}{\tilde{N}^*} > 1$.

Since $\tilde{N}^{no}/\tilde{N}^* = \frac{(v-q)/p}{M} = \frac{(v-q)/p}{r_u}$, it is obvious that the value of $\tilde{N}^{no}/\tilde{N}^*$ decreases with r_u, *i.e.*, RLV.

When $A < 0$, the relationship is reversed. ■

From literatures [9], [10], we know that the SDPs have the positive effect of attracting users. This is true when the market consists of the negative-cost CPs, which have high per unit revenue and low rate of traffic consumption, like *Google*

Search. However, when the market consists of positive-cost CPs, the SDPs enforce the competition and increase the operating cost to CPs simultaneously. Finally, more CPs exit the market. Nevertheless, when users prefer a greater RLV, the gap between the equilibrium number of CPs in the market with and without SDPs is reduced.

B. Heterogeneous Content Providers

Different from the first scenario, in this subsection, we focus on analyzing the behaviors of heterogeneous CPs, in which CPs differ from each other in α_i and v_i. Specifically, CP_i with a larger v_i has potential to sponsor more so as to obtain more competitive advantages. Clearly, α_i depends on the type of contents, *e.g.*, video as compared to email. α_i can also be considered as an indicator of the technology of CP_i, especially for the same type of contents. Considering two video CPs CP_i and CP_j with same per unit revenue, they provide the same content for users. A smaller α_i may mean that CP_i has advanced video coding technology of transmitting the same video in a smaller traffic volume, thus CP_i has more competitive advantages over CP_j. And we will further analyze how these factors affect the competition.

1) The Short-Run Equilibrium: We start from the short-run scenario and study how SDP and RLV affect the competition among heterogeneous CPs. To this end, we first derive the market equilibrium. When the set of CPs $\tilde{\mathcal{N}}$ is given, the market equilibrium should satisfy the following conditions:

i) Each user maximizes his or her utility subject to the data cap constraint;

ii) No CP can increase its profit by unilaterally changing its sponsored level.

Lemma 4: If the conditions (11) (12) and (14) are satisfied, there exists a unique λ such that the market is in the equilibrium.

Proof of Lemma 4: Considering the left side of users' constraint, denoted as Λ, we have

$$\Lambda = \sum_{i=1}^{N} \alpha_i A_i t_i/(1 - r_u(t_i)).$$

Then, it follows

$$\begin{aligned}\frac{\partial \Lambda}{\partial t_i} &= \alpha_i A_i \frac{1 - r_u + t r_u'}{(1 - r_u(t_i))^2}\\ &= \alpha_i A_i \frac{1 + (r_u)^2 - r_u r_{u'}}{(1 - r_u(t_i))^2}\\ &> \alpha_i A_i \frac{1 + (r_u)^2 - 2r_u}{(1 - r_u(t_i))^2} > 0.\end{aligned}$$

Note that

$$u'(t_i)[1 - r_u(t_i)] = \tilde{\lambda} \alpha_i A_i.$$

Let $\psi(t_i) = u'(t_i)[1 - r_u(t_i)]$. Since $\psi'(t_i) = (2 - r_{u'}(t_i))u''(t_i) < 0$, then $\psi(t_i)$ is strictly decreasing with t_i. Then, we can know that t_i is strictly decreasing with λ. It follows that Λ is strictly decreasing with λ. Thus, there exist only one unique solution for optimal λ. ■

This lemma guarantees the uniqueness of equilibrium λ in the short-run market. Combined with Lemma 3, each CP's optimal sponsored level can be uniquely determined. We now study how heterogeneous CPs differ their strategies under the optimal decisions by the following theorems.

Theorem 3 (Differentiated Subsidy): In the short-run market, for any CP_i and CP_j, where $i, j \in \tilde{\mathcal{N}}$, the sponsored level in the equilibrium satisfies

i) *If $\alpha_i = \alpha_j$ and $v_i > v_j$, then $h_i > h_j$;*

ii) *If $\alpha_i < \alpha_j$ and $v_i = v_j$, then $h_i > h_j$ when $r_u' < 0$, and $h_i < h_j$ when $r_u' > 0$.*

Proof of Theorem 3: Recalling from Equations (8) and (10), under the equilibrium $\tilde{\lambda}$, we have first-order condition of ϕ_i respects to t_i as

$$u'(t_i)[1 - r_u(t_i)] = \tilde{\lambda} \alpha_i A_i.$$

Let $\psi(t_i) = u'(t_i)[1 - r_u(t_i)]$. Since $\psi'(t_i) = (2 - r_{u'}(t_i))u''(t_i) < 0$, then $\psi(t_i)$ is strictly decreasing with t_i. Thus, for any $0 < \alpha_i A_i < \alpha_j A_j$, we have

$$\frac{\psi(t_i)}{\psi(t_j)} = \frac{\alpha_i A_i}{\alpha_j A_j} < 1. \tag{20}$$

The above equation implies that $t_i > t_j$.

Since $\bar{h}_i = u'(t_i)/(\lambda \alpha_i)$ and $u'(t_i)$ decreases in t_i, thus the condition of $t_i > t_j$ implies that $\alpha_i \bar{h}_i < \alpha_j \bar{h}_j$. In other words, the CP_i with smaller cost will let its users take less bandwidth cost. In particular, if CP_i has higher bandwidth requirement, *i.e.*, $\alpha_i \geq \alpha_j$, the sponsoring proportion should be higher. The above equation is also equivalent to

$$\frac{\bar{h}_i/A_i}{\bar{h}_j/A_j} = \frac{1 - r_u(t_j)}{1 - r_u(t_i)}. \tag{21}$$

From Equation (20), we have $\frac{u'(t_i)}{u'(t_j)} = \frac{\alpha_i A_i}{(1 - r_u(t_i))} \frac{(1 - r_u(t_j))}{\alpha_j A_j} = \frac{\alpha_i \bar{h}_i}{\alpha_j \bar{h}_j}$. Since $u''(t) < 0$, then $u'(t)$ decreases in t, thus $u'(t_i) < u'(t_j)$ for $t_i > t_j$, so we have $\frac{u'(t_i)}{u'(t_j)} < 1$, that is $\alpha_i \bar{h}_i < \alpha_j \bar{h}_j$. When $\alpha_i = \alpha_j$, then $\bar{h}_i < \bar{h}_j$, that is $h_i > h_j$.

From Equation (21), if $r_u' > 0$, then $r_u(t_i) > r_u(t_j)$ for $t_i > t_j$, thus $\bar{h}_i/A_i > \bar{h}_j/A_j$, and vice versa. When $v_i = v_j$ and $r_u' > 0$, then $\bar{h}_i > \bar{h}_j$, that is $h_i < h_j$, and vice versa. ■

In this theorem, $\alpha_i = \alpha_j$ and $v_i > v_j$ indicates that CP_i and CP_j are of similar type of contents, *e.g.*, all videos, yet CP_i has a greater per unit revenue as compared to CP_j. In such situation, CP_i will sponsor more. Also in this theorem, $\alpha_i < \alpha_j$ and $v_i = v_j$ indicates that the CP_i has better technology level and the same per unit revenue. In such situation, the sponsored level is dependent with the market types, that is, $h_i > h_j$ in the variety-avoider market and $h_i < h_j$ in the variety-lover market.

Next we study the competition of CPs with and without SDPs. Let ϕ_i^{no} be the utility of CP_i where the market has not adopted the SDPs.

Theorem 4 (Market Fairness): In the short-run market with constant RLV ρ (or ρ'), for any CP_i or CP_j, where $i, j \in \tilde{\mathcal{N}}$ such that $0 < \alpha_i A_i < \alpha_j A_j$, we have the following results in the equilibrium,

i) $\frac{\phi_i}{\phi_j} = \frac{\phi_i^{no}}{\phi_j^{no}}$ *if $v_i = v_j$, and* $\frac{\phi_i}{\phi_j} > \frac{\phi_i^{no}}{\phi_j^{no}}$ *if $\alpha_i = \alpha_j$;*

ii) *For any $\rho < \rho'$, we have* $\frac{\phi_i}{\phi_j} > \frac{\phi_i'}{\phi_j'} > 1$.

Proof of Theorem 4: Considering the CP_i's utility after sponsorship, we have $\phi_i(t_i) = \frac{r_u}{1-r_u}\alpha_i A_i pLt_i - s = u''(t_i)t_i^2 pL/\lambda - s$. Here, we ignore the cost s and set it as zero. Then, we have $\frac{\phi_i}{\phi_j} = \frac{u''(t_i)t_i^2}{u''(t_j)t_j^2} = |\frac{u''(t_i)t_i^2}{u''(t_j)t_j^2}|$. Note that when $0 < \alpha_i A_i < \alpha_j A_j$, we have $t_i > t_j$. Define $g(t) = u''(t)t^2$ (note that $u''(t)t^2 < 0$). Then, we have $g'(t) = tu''(2 - r_{u'}) < 0$. Thus, we have $|g(t_i)| > |g(t_j)|$, which implies that $\phi_i > \phi_j$.

In particular, for constant RLV (*i.e.*, $u(t) = \frac{t^{1-\rho}}{1-\rho}$), we have $t_i = \left(\frac{\lambda\alpha_i A_i}{1-\rho}\right)^{-1/\rho}$, $\frac{\phi_i}{\phi_j} = \left(\frac{\alpha_i A_i}{\alpha_j A_j}\right)^{1-1/\rho}$.

Considering the CP_i's utility before sponsorship, we have $\phi_i(t_i) = (v_i - q)\alpha_i t_i L - s$. If we ignore s, we have $\frac{\phi_i^{no}}{\phi_j^{no}} = \frac{v_i-q}{v_j-q}\left(\frac{\alpha_i}{\alpha_j}\right)^{1-1/\rho}$. To compare $\frac{\phi_i}{\phi_j}$ and $\frac{\phi_i^{no}}{\phi_j^{no}}$, we only need to compare $\frac{\phi_i/\phi_i^{no}}{\phi_j/\phi_j^{no}} = \frac{(p+q-v_i)^{1-1/\rho}/(v_i-q)}{(p+q-v_j)^{1-1/\rho}/(v_j-q)}$. When $v_i = v_j$, then $\frac{\phi_i/\phi_i^{no}}{\phi_j/\phi_j^{no}} = 1$, that is $\frac{\phi_i}{\phi_j} = \frac{\phi_i^{no}}{\phi_j^{no}}$.

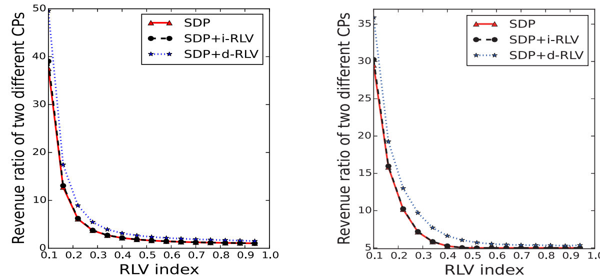
When $\alpha_i = \alpha_j$, we have $v_i > v_j$. Let $h(v) = (p + q - v)^{1-1/\rho}/(v - q)$, then, we have $h'(v) = \frac{(p+q-v)^{-1/\rho}}{(v-q)^2}[-p + (v-q)/\rho]$. When $v > p\rho+q$, $h(v)$ is increasing function, then we have $\frac{\phi_i/\phi_i^{no}}{\phi_j/\phi_j^{no}} > 1$. When $v < p\rho + q$, $h(v)$ is decreasing function. Then, we have $\frac{\phi_i/\phi_i^{no}}{\phi_j/\phi_j^{no}} < 1$. Note that $A/(1 - r_u) \leq 1$, then we have $v > p\rho + q$. That means, $\frac{\phi_i/\phi_i^{no}}{\phi_j/\phi_j^{no}} > 1$, *i.e.*, $\frac{\phi_i}{\phi_j} > \frac{\phi_i^{no}}{\phi_j^{no}}$.

When $\rho < \rho'$, we have $\frac{\phi_i'}{\phi_j'} = (\frac{\alpha_i A_i}{\alpha_j A_j})^{1-1/\rho'} = (\frac{\alpha_j A_j}{\alpha_i A_i})^{1/\rho'-1} > 1$. To compare $\frac{\phi_i}{\phi_j}$ and $\frac{\phi_i'}{\phi_j'}$, we only need to compare $\frac{\phi_i/\phi_i'}{\phi_j/\phi_j'} = (\frac{\alpha_i A_i}{\alpha_j A_j})^{1-1/\rho} \cdot (\frac{\alpha_j A_j}{\alpha_i A_i})^{1-1/\rho'} = (\frac{\alpha_j A_j}{\alpha_i A_i})^{1/\rho-1/\rho'} > 1$, *i.e.*, $\frac{\phi_i}{\phi_j} > \frac{\phi_i'}{\phi_j'} > 1$. Then, we complete the proof. ■

This theorem considers CP_i and CP_j in the market with constant RLV, where the content of CP_i incurs a smaller cost as compared to CP_j. The first part of this theorem shows when the two CPs have the same profitability, SDPs will not increase the differences of their revenue. It also states that a CP with a higher revenue always has a larger difference via SDPs. In other words, the market becomes more unfair. The second part of this theorem shows that the advantage of big CP_i under SDPs is reduced as users prefer larger RLV (*i.e.*, the gap of CP_i and CP_j becomes smaller when ρ increases).

To further confirm the conclusions of Theorem 4, we have provided more simulation results, which focus on the second part of Theorem 4. Fig. 6 shows the impact of the greater RLV, where two CP_i and CP_j only have different rates of traffic consumption, *i.e.*, $\alpha_i \neq \alpha_j$, or different per unit revenues, *i.e.*, $v_i \neq v_j$. Both Fig. 6(a) and Fig. 6(b) show that the revenue gap of the two CPs decreases quickly with the increasing of RLV index, that is, the advantage of big CP_i under SDPs is reduced as users prefer larger RLV.

2) The Long-Run Equilibrium: Next, we study the impact of user quantity on the long-run equilibrium under the variety preference. For the CPs with the same type, let z and $\Gamma(z)$ be the random variable of types and the distribution of z over $\mathcal{N}$, respectively. To derive the equilibrium, we assume that there exists a cutoff cost $\bar{z}$ such that for any CP_i, if $z_i < \bar{z}$, CP_i will stay in the market. Otherwise, it will leave the market. Then, we have following lemma.



(a) $\alpha_i \neq \alpha_j$ and $v_i = v_j$ (b) $\alpha_i = \alpha_j$ and $v_i \neq v_j$

Fig. 6. Impact of greater RLV on the advantage of big CP.

Lemma 5: If conditions (11) (12) and (14) are satisfied, there exists a unique pair $(\bar{z}, N)$ in the market equilibrium.

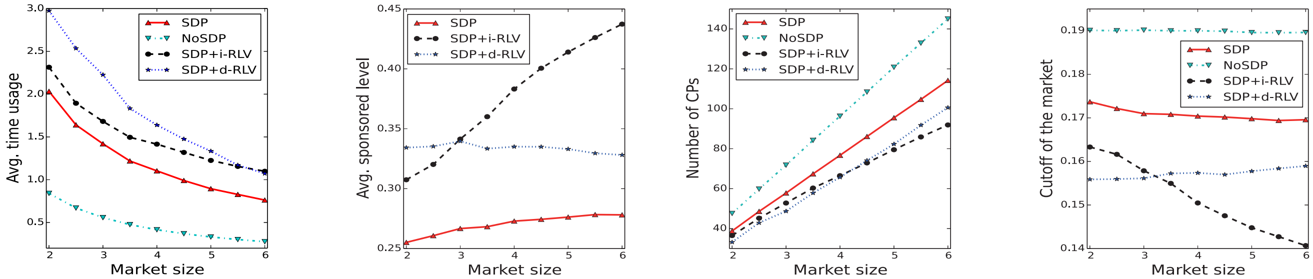
Proof of Lemma 5: Due to the complexity of the proof, the details can be found in our online technical report [44]. Similarly, proofs of Theorem 5, Theorem 6, and Theorem 7 can also be found in our online technical report [44]. ■

This lemma guarantees the uniqueness of the market equilibrium. Then, we can analyze the impact of user quantity on the long-run equilibrium.

Theorem 5 (User Quantity Effect): In the long-run market, if $r' > 0$ ($r' < 0$), then the cutoff cost decreases (increases) with L and the sponsored level increases (decreases) with L.

To provide a clearer understanding about Theorem 5, some simulation results can be found in Fig. 7. When the user quantity becomes larger, the average time usage always decreases, as shown in Fig. 7(a). This is because each CP can earn more revenue and thus attracts more CPs entering the market, as shown in Fig. 7(c). This has different effects on different markets. Specifically, when the users are variety-lovers (*i.e.*, $r' > 0$), due to the user's traffic consumption for each individual CP is declining, the RLV becomes smaller, which leads to more intense competition in the market. The CPs need to sponsor more so as to survive in the market, as shown in Fig. 7(b). It triggers the exiting of the higher-cost CPs (*i.e.*, cutoff cost decreasing), as shown in Fig. 7(d). On the contrary, Fig. 7(b) and Fig. 7(d) also show that when users are variety-avoiders, a large market will weaken the competition, thus the average sponsored level decreases. Meanwhile, the variety-avoider market has lower cost of entering the market for CPs, indicating that more CPs with higher-cost can survive in the market (*i.e.*, cutoff cost increasing).

In Summary: Through our analysis of the competition among CPs, we once again prove some existing results of previous studies when we take variety into consideration. For example, under the influence of SDPs, the number of CPs in the market decreases if each CP has positive cost (Theorem 2) and big CPs (*i.e.*, CPs with higher per unit revenue) have advantage over small CPs (Theorem 3). But, the influence of SDPs will be reduced if users have a greater RLV (Theorem 2, Theorem 4). Furthermore, under SDPs, the advantages of CPs with better technology may decrease when users have higher variety demand (Theorem 4), which



(a) Time usage vs. L (b) Sponsored level vs. L (c) Number of CPs vs. L (d) Market cutoff vs. L

Fig. 7. Impact of L on the long-run market.

shows that the excessive RLV will cut down the benefit of technology as well. In addition, we get some new results. After taking variety into consideration, we find that the CP quantity affects the sponsored level (Theorem 1). In previous studies, it is previously believed that the CP quantity is independent from the sponsored level. And we find that both the sponsored level and the number of CPs are affected by the user quantity (Theorem 5), which is a completely new result that has not been discovered in previous studies.

V. Impact of ISP's Strategy

Although SDPs enables CPs to influence market pricing, the ISP remains the dominant player in the wireless data market. To study the monopolistic ISP's best strategy and its impact on the market, we first analyze the short-run market where the $(\mathcal{N}, \tilde{\mathcal{N}}, \mu, L)$ keeps unchanged. After that, we further analyze the long-run market where the $(\mathcal{N}, \tilde{\mathcal{N}}, \mu, L)$ can be changed. In addition, we analyze the market with homogeneous CPs at first and then carry out the evaluation to analyze the market with heterogeneous CPs.

A. The Short-Run Market

In the short-run market, there exists a fixed number of CPs. These CPs' optimal decisions are significantly affected by the ISP's strategy, and thus affects users' time usage. Based on the best responses of CPs and users, the ISP decides its optimal strategy to maximize its revenue. We first consider the homogeneous market and derive the following theorem.

Theorem 6 (Short-run Impact): If $t < \hat{t}$ in the short-run equilibrium, the impact of ISP's strategy satisfies

i) $\frac{\partial t}{\partial p} < 0$ *and* $\frac{\partial t}{\partial H} > 0$;

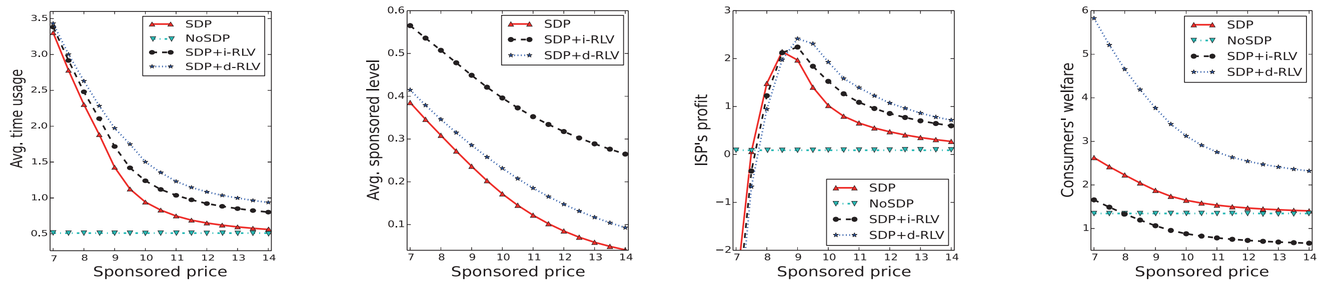
ii) $\frac{\partial h}{\partial p} < 0$ *and* $\frac{\partial h}{\partial H} > (<)\ 0$ *when* $r'_u < (>)\ 0$.

Theorem 6 states that both users' time usage and CPs' sponsored level decrease with the sponsored price. When the ISP increases the data cap, users' time usage always increases until it reaches the maximum value. However, the CPs' sponsored level depends on users' RLV categories. The intuition is that a larger data cap can make users prefer a smaller RLV in the variety-avoider market. The CPs can be substituted more easily and thus the competition becomes more intense. In particular, when the market belongs to the variety-free category, the CPs' sponsored level is independent with ISP's data cap.

We now use simulations to understand the short-run market with heterogeneous CPs. We consider the market with $N = 100$ CPs and one ISP to explore the key features of the market. The per unit revenue of each CP is randomly selected from $[\$1, \$10]$ [5], [7]. The rate of traffic consumption of each CP is randomly selected from $[0.05, 0.5]$ (GB/hour), *e.g.*, watching online movies on smartphone through 4G may consume the volume of 350MB traffic per hour [45]. We adopt user's utility function in Equation (5), with the parameter ρ for each CP randomly distributed over $[0.2, 0.8]$ and $(a, b) = (1, 0)$ for variety-lovers and $(a, b) = (0, 1)$ for variety-avoiders. We set user's maximum consumption time for one CP in the scope of $[1h, 20h]$ [46]. CP's connection service fee and user's data cap are set as \$1/GB and 10GB [47], respectively. We adopt the capacity sharing congestion function and let the congestion level fee be $\chi = 10$ and the load sensitivity be $\delta = 3$. Note that our simulations do not depend on particular settings, and our purpose is to show qualitative trends in general.

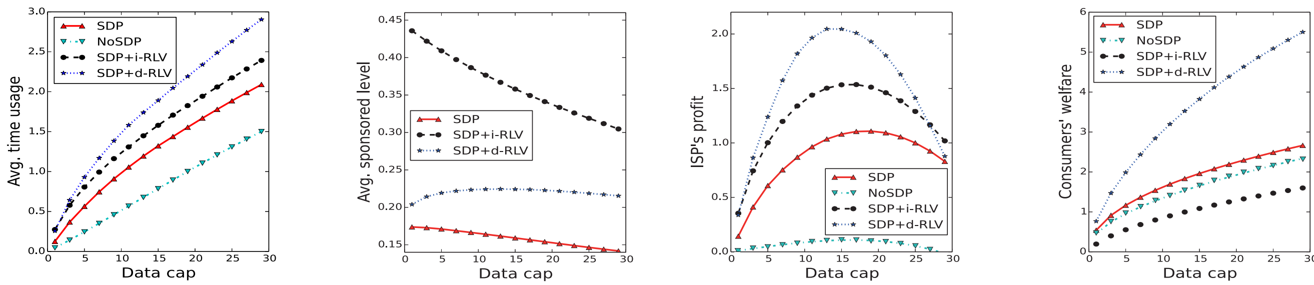
Through our experiments, the impact of ISP's strategy (p, H) on the short-run market has been illustrated in Fig. 8 and Fig. 9. Specifically, the impact of sponsored price can be found in Fig. 8. As shown in Fig. 8(a) and Fig. 8(b), the average time usage and the sponsored level always decrease with p increasing. And Fig. 8(c) shows that suitable p is required for ISP, *e.g.*, $p = 8.5$ can maximize ISP's profit under the variety-free markets. Fig. 8(d) shows that the consumers' welfare always decreases with p increasing since the higher sponsored price limits user's traffic usage.

From Fig. 9(a), we can see when the ISP increases H, users' time usage always increases. And different lines show that the SDP can further increase user's time usage. Fig. 9(b) shows that the sponsored level always has a decreasing trend even for variety-free markets, which may be contrary to Theorem 6. The intuitive behind is that the maximum time usage for some CPs are approached. Higher data cap usually means the competition among CPs becomes more moderate. Each user can approach its maximum time usage by sponsored less. Even under the time usage constraint, the sponsored level still increases with data cap under the variety-avoider market, especially when data cap is small. Fig. 9(c) shows that the optimal H for ISP is different under different market. Fig. 9(d) shows that users' welfare always increases with H increasing. In addition, both Fig. 9(c) and Fig. 9(d) show that the ISP and users always benefit from SDPs, especially when users are variety-avoiders.



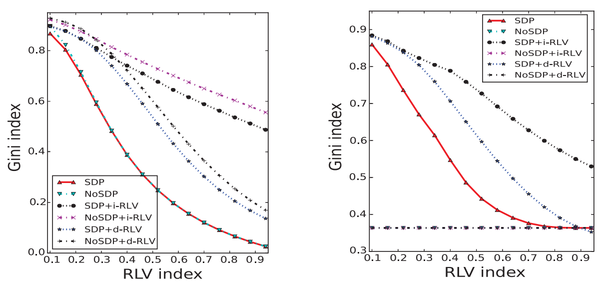
(a) Time usage vs. p (b) Sponsored level vs. p (c) ISP's profit vs. p (d) Consumer welfare vs. p

Fig. 8. Impact of p on the short-run market.



(a) Time usage vs. H (b) Sponsored level vs. H (c) ISP's profit vs. H (d) Consumer welfare vs. H

Fig. 9. Impact of H on the short-run market.



(a) $\alpha_i \neq \alpha_j$ and $v_i = v_j$ (b) $\alpha_i = \alpha_j$ and $v_i \neq v_j$

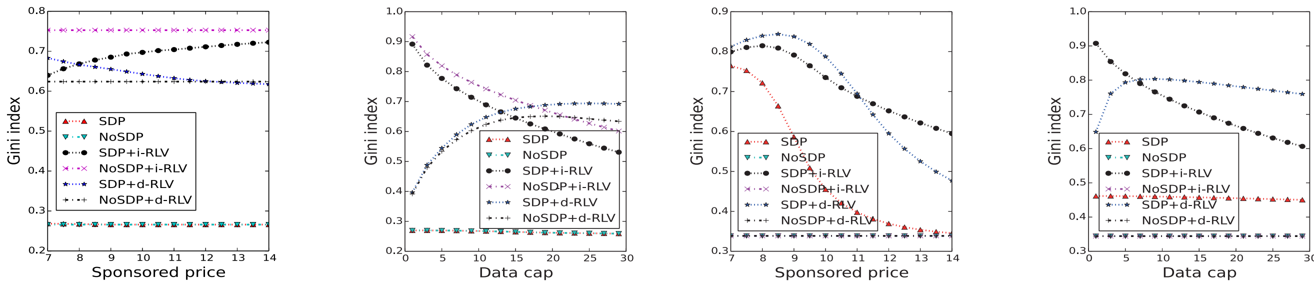
Fig. 10. Impact of ρ on the Gini index in short-run market.

In addition to the direct impact of ISP's strategy, we adopt widely known metric, Gini index [48], to measure the market fairness. Higher Gini index indicates smaller fairness. The Gini index equals to 0 implying extreme fairness while the Gini index equals to 1 implying extreme unfairness. Fig. 10 shows the impact of variety indicator ρ on the market fairness. Both Fig. 10(a) and Fig. 10(b) show that users would prefer a larger RLV, because the market becomes more fair. Fig. 10(a) shows that when CPs only have different rates of traffic consumption, *i.e.*, $\alpha_i \neq \alpha_j$, the fairness gap between the market with SDPs and the market without SDPs may keep the same, or becomes larger. Fig. 10(b) shows that when CPs only have different per unit revenues, *i.e.*, $v_i \neq v_j$, the fairness of the market with SDPs approaches that of the market without SDPs.

Fig. 11(a) and Fig. 11(b) show the fairness, where CPs only have different rates of traffic consumption, *i.e.*, $\alpha_i \neq \alpha_j$, which can be considered as an indicator, reflecting the technology level of a CP. Generally, it is better for the market to encourage the unfairness caused by technical difference, because an unfair market can encourage CPs to improve the technology and reduce the required bandwidth. Fig. 11(a) shows that SDP cannot always increase the unfairness in the market, which only happens when users are variety-avoiders (the line of SDP+d-RLV is above that of NoSDP+d-RLV). When users are variety-lovers, SDP makes the market more fair. It also shows that higher sponsored price makes smaller difference of fairness between the market with SDP and that without SDP. Fig. 11(b) illustrates the market becomes more unfair (fair) with increasing of H when users are variety-avoiders (variety-lovers). In addition, the fairness gap between the market with and without SDP becomes larger. Fig. 11(c) and Fig. 11(d) show the fairness, where CPs only have different per unit revenue, *i.e.*, $v_i \neq v_j$. Fig. 11(c) illustrates that SDP always makes the market more unfair. This may result in an unhealthy market since the market prefers the rich CPs if the SDPs are adopted. Fortunately, the unfairness can be alleviated when the sponsored price is higher. In addition, when users are variety-lovers, the market also becomes more fair as the ISP enlarges its data cap, as shown in Fig. 11(d). However, when users are variety-avoiders, the unfairness may increase.

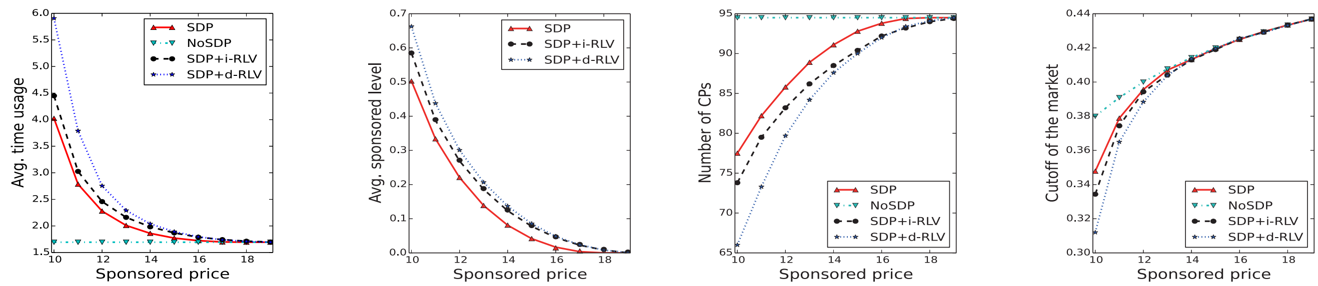
B. The Long-Run Market

In the long-run market, the ISP can improve its capacity so as to reduce the congestion cost via building more base stations, and deploying advanced technology. However, these improvements in infrastructures require financial support. The ISP have to handle the trade-offs on pricing and performance. And then, the monopolistic ISP's strategy can affect the revenue of CPs. Once the revenue has not reached expectations for a long time, CPs have the right to withdraw from the market. The incumbent number of CPs $\tilde{N}$, therefore, is a



(a) $\alpha_i \neq \alpha_j$ and $v_i = v_j$ (b) $\alpha_i \neq \alpha_j$ and $v_i = v_j$ (c) $\alpha_i = \alpha_j$ and $v_i \neq v_j$ (d) $\alpha_i = \alpha_j$ and $v_i \neq v_j$

Fig. 11. Impact of (p, H) on the Gini index in short-run market.



(a) Time usage vs. p (b) Sponsored level vs. p (c) Number of CPs vs. p (d) Market cutoff vs. p

Fig. 12. Impact of p on the long-run market.

variable instead of a constant. In other words, the ISP's strategy can affect the equilibrium number of CPs in the market. To thoroughly analyze these issues, we first consider the market with homogeneous CPs, and the impact of such capacity extension can be obtained by the following theorem.

Theorem 7 (Long-run Impact): In the long-run market, we consider the equilibria in $(\mathcal{N}, \tilde{\mathcal{N}}, \mu, L)$ and $(\mathcal{N}, \tilde{\mathcal{N}}', \mu', L)$ two systems. If $\mu < \mu'$, then we have

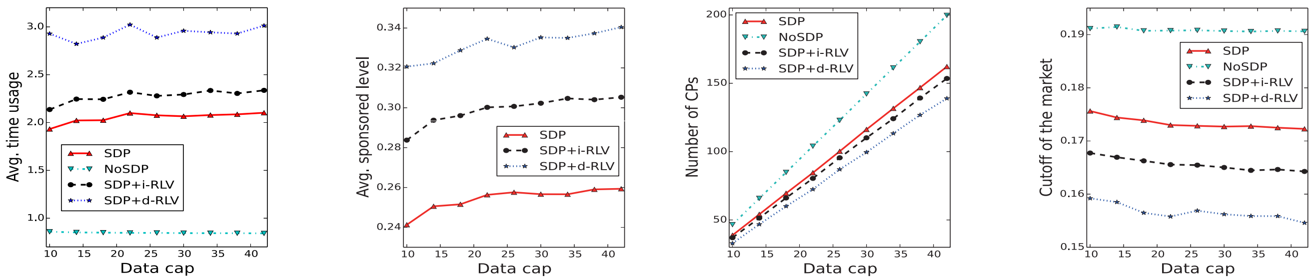
i) *The ISP's strategy satisfies $H \leq H'$ and $p \geq p'$;*
ii) *The number of CPs satisfies $\tilde{N} \leq \tilde{N}'$.*

Theorem 7 states that when the ISP expends its capacity, the ISP's optimal p is reduced and H is increased. This will increase the revenue of CPs, thus the market can accommodate more CPs. It will facilitate the competition, which leads to higher sponsored level and user's traffic usage. This partially counteracts the effect of capacity expansion.

To further analyze the impact of ISP's strategy (p, H) on the long-run market, we also provide some simulation experiments, in which most of the basic parameters are the same with the short-run market. Fig. 12 illustrates the impact of sponsored price on the long-run market. It can be found from Fig. 12(a) and Fig. 12(b) that, the average time usage and the sponsored level always decrease with p increasing, which is consistent with the short-run market. And it reduces the burden of each CP due to the sponsored strategy. Thus, each CP's revenue increases, which results in more CPs in the market, as shown in Fig. 12(c). Fig. 12(d) shows that the cutoff of the market always increases with p increasing, especially under SDP. This indicates that the requirement of the market decreases and more CPs with higher cost can enter the market.

Different from the short-run market, the average time usage and the average sponsored level both increase slightly in the long-run market, as shown in Fig. 13(a) and Fig. 13(b). The reason behind this is that the number of CPs in the market increases a lot, as shown in Fig. 13(c), which counteracts the effects of traffic cap increasing. Due to the operating costs of CPs increasing more slightly, the requirement to enter the long-run market only has a slight change, as shown in Fig. 13(d). It also shows that SDP improves the requirement to enter the long-run market. In addition, Fig. 12(c) and Fig. 13(c) also show that the SDP reduces the number of CPs in the market since it improves the requirement to enter the long-run market.

In summary: Through our analysis, it has been found that the influence of ISP's strategy still cannot be neglected when we consider the variety demand, and we get some new results. In the short-run market, if the ISP increases the sponsored price, both user traffic usage and the sponsored level of CPs decreases. And if the ISP increases the data cap, the user traffic usage increases. These conform to the conclusions of previous studies. But the sponsored level depends on market variety. And for a variety-lover market, a greater data cap will lead to a smaller sponsored level. The surprising result is that for a variety-avoider market, a greater data cap may lead to an increase in the sponsored level, *i.e.*, the competition among CPs becomes tougher. Intuitively, this is because the increased traffic usage does not lead to a matched increase in variety. Thus, the competition among CPs has intensified. In the long-run market, the number of CPs in the market increases if the ISP increases the data cap. The increasing number of CPs in the market counteracts data cap increase. As a result, the sponsored level of CPs and users traffic usage increase slightly as compared with the short-run market. Meanwhile, if the user quantity becomes larger, the number of



(a) Time usage vs. H (b) Sponsored level vs. H (c) Number of CPs vs. H (d) Market cutoff vs. H

Fig. 13. Impact of H on the long-run market.

CPs increases. In such situation, when users are variety-lovers the requirement to enter a market increases, and vice versa.

VI. Obtaining Market Parameters

Some parameters in our model are not directly available for the ISPs. In this section, we develop a method to calibrate these parameters through collectable data in the market, which can also be applied to quantity the love of variety. It can help the ISPs analyze SDPs more thoroughly, and further promote the deployment of SDPs. And then, a large amount of wireless data will be consumed per day, providing more data for machine learning-based applications [49], [50].

In our model, Equation (5) is one of the important factors to define the RLV. However, parameters ρ, a, b of Equation (5) are abstract and cannot be directly obtained from the data in the market. In real-world deployments, what an ISP can collect are the sample values of t_{li}, where t_{li} refers to the time that user l spent on a CP_i. Let Y_{li} be the sample values of t_{li}. Based on Y_{li}, we propose a least square method to calibrate ρ, a, b as follows.

Since the users can adjust their traffic consumption time t_{li} to maximize their utility, Y_{li} can be considered as the sample of user l's optimal decision for CP_i. Therefore, we can use the optimal user's decision (*i.e.*, Equation (15)), to achieve the calibration. In this equation, we have four unknown parameters ρ, a, b and λ. Because both α_i and A_i, which is determined by p, q, v_i, can be obtained from the data in the market. Note that ρ, a, b are independent from each other. λ, on the other hand, is a dependent parameter of ρ, a, b. To this end, we first compute ρ, a, b and then compute λ.

Using the least squares method, we compute the sum of the least squares errors between the observed data and theoretical parameters. As such, we obtain a set of residual sum of squares equations (RSS). After that, we take the partial derivatives of these *RSS* equations with respect to ρ, a, b. The values of ρ, a and b can be solved by setting these equations to 0. More specifically, these equations are

$$\begin{cases} \sum_{i=1}^{\tilde{N}} (Y_{li} - \nu^{-\frac{1}{\rho}} + a)\nu^{-\frac{1}{\rho}} \ln \nu = 0, \\ \sum_{i=1}^{\tilde{N}} (\nu^{-\frac{1}{\rho}} - Y_{li} - a)\nu^{-\frac{1}{\rho}-1} = 0, \\ a = -\dfrac{\sum_{i=1}^{\tilde{N}} (Y_{li} - \nu^{-\frac{1}{\rho}})}{N}, \end{cases} \tag{22}$$

where $\nu = \lambda\alpha_i - b$.

Based on these results, we compute the λ by an iterative algorithm, as shown in Algorithm 1. The idea of the algorithm is to first set an initial value of λ and compute ρ, a, b using the least squares method. Then we iteratively recompute λ by Equation (23) until convergence, *i.e.*,

$$\lambda = \frac{\sum_{i=1}^{\tilde{N}} t_i[(a+t_i)^{-\rho} + b]}{H}. \tag{23}$$

Algorithm 1 An Iteration Algorithm to Calculate λ

Input: The number of CPs, and initial λ_0

```
Initialize the value of λ as λ = λ0
while the value of λ is not convergent do
    for each sample value Y_li do
        Calculate ρ, a, b with Equation (22)
    Calculate new λ by the new ρ, a, b
    Updating λ by Equation (23)
Return λ
```

The above method provides a general approach for both homogenous users with a uniform set of ρ, a, b, and heterogeneous users with different ρ_l, a_l, b_l for each user l. Once the specific values of these parameters are obtained, we can combine the relevant definitions in Section III to quantify some of the indicators, *e.g.*, RLV and SRE. For example, the RLV can be evaluated with Equations (1) and (5). Note that it is possible to use more specific parameter calibration methods when we can obtain certain pre-knowledge of the market [51], [52]. For example, if we have sample values of the user's utility, we can apply the Kmenta approximation [51], which is mainly based on Taylor's expansion and nonlinear least squares methods. Bayesian approaches [52] can also be applied where they commonly have higher accuracy and require fewer observation values.

VII. Conclusion

Previous studies, which are used to understand what SDPs will bring about to the market regarding the only model price as the driving factor for user decision optimization. We argue that they overlook the content variety demand of users. Therefore, we develop a new model to study the competition among CPs under SDPs in this paper. In order to integrate such variety into our new model, we define RLV as an index. Through a series of transformation, we integrate RLV into an overall two-stage Stackelberg game model. We conduct a comprehensive analysis on the competition among CPs, then derive a set of results out of our new model, some of which are consistent with (or rectify) previous studies, and some of which are even completely new results, that is, they haven't appeared in previous studies. To make our new model more practical, we further develop a method to calibrate the abstract parameters, which bridges the gap between the model and analyzing a practical market under SDPs. Overall, the new model proposed in this paper further understands the impact of SDPs on the practical market, and is conducive to stakeholders making decisions in SDP ecosystem.

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