Congestion-Aware Modeling and Analysis of Sponsored Data Plan from End User Perspective

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***Abstract*—The past decade has witnessed the rapid expansion of demands for mobile traffic, while the traditional mobile traffic pricing schemes cannot accommodate such demands. Sponsored data plan (SDP), which can increase the revenue of all stakeholders in the market through transferring some of the revenue from content providers (CPs) to end users (EUs), is more suitable. However, existing studies have focused more on Internet service providers (ISPs) and CPs, ignoring the influence of EUs (*e.g.*, the inherent attribute differences of EUs and the interaction among EUs) on the market under SDP. Regarding the difficulty of modeling the abstract property about interaction among EUs, we utilize network congestion as the medium and construct the congestion-aware SDP model based on Stackelberg game. The newly proposed model can not only analyze how network congestion affects SDP mechanism, but also elucidate the impact of interactions among EUs. More specifically, through theoretical analysis, we prove that there is a unique dynamic equilibrium in the interaction among EUs (*i.e.*, the traffic consumption of different EUs). By taking into account network congestion, the newly proposed model also more accurately and realistically describes the optimal strategies and computation methods of all stakeholders in the market. Moreover, simulation experiments demonstrate that the positive effect brought by SDP is not as obvious as before, and EUs influence each other instead of being independent of each other. Overall, this paper emphasizes the non-negligible influence of EUs and promotes a deeper understanding of SDP mechanism, which can guide the relevant stakeholders to optimize their own decision-making details.**

***Index Terms*—Network Congestion, SDP, Revenue, Link Utilization Rate, Interaction among EUs.**

I. Introduction

With the rapid development of intelligent mobile terminals and mobile communication technologies (*i.e.*, LTE and 5G), mobile networks become the indispensable approach for end users (EUs) to access the Internet. Cisco has forecasted that smart phone traffic will exceed PC traffic, and traffic from wireless and mobile devices will account for more than 71% of the total IP traffic by 2022 [1]. This causes that the advertising revenue of content providers (CPs) is mainly from mobile networks, rather than traditional wired networks. However, the traditional mobile traffic pricing methods cannot allow Internet service providers (ISPs) and EUs to benefit from advertising revenue. In contrast, Sponsored data plan (SDP) not only enables more reasonable pricing, but also allows ISPs and EUs to benefit from the increased revenue of CPs. Therefore, SDP is widely supported by the industry and attracts the interest of scholars.

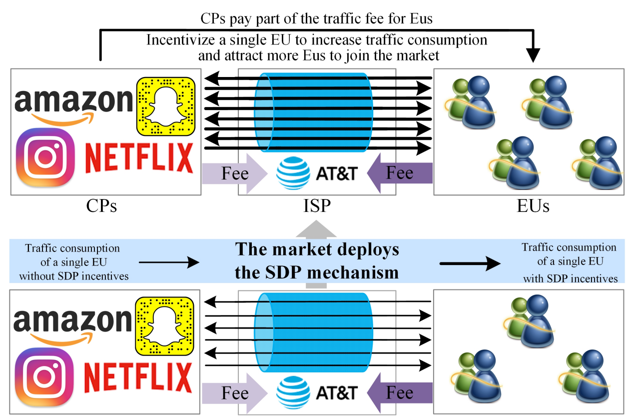

Fig. 1. The relationship between stakeholders in the mobile Internet market, and the difference in link utilization rate caused by SDP. Note that, the difference before and after deploying SDP is thickness of the line. For the same link capacity, more and thicker line means more network congestion (*i.e.*, tighter interaction among EUs).

Fig. 1 succinctly summarizes the relationship between these stakeholders (*i.e.*, ISPs, CPs, and EUs) in the mobile Internet market under SDPs. More specifically, through allowing CPs to subsidize partial or all the traffic expense for their EUs, SDP is utilized to transfer the revenue from CPs to EUs, reducing the EUs' financial expense and promoting more traffic consumption (*i.e.*, more traffic revenue of ISPs and more

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advertising revenue of CPs), and then creating a tripartite (*i.e.*, ISPs, CPs, and EUs) win-win phenomenon. Overall, SDP can promote the prosperity of the market.

Since SDP was put forward by AT&T [2], it has produced a series of related challenges, attracting a large number of research interests. To better understand the mechanism of SDP, existing studies [3]–[5] mainly focus on the basic principles of ISPs or CPs, such as how ISPs should specify the price, and how CPs should set the subsidy ratio. In addition, there are some more in-depth studies [6], [7] with ISPs or CPs, such as in the SDP market, whether there will be fairness problem between CPs of different sizes? For example, might large-scale CPs be more advantageous? Will SDP exacerbate competition among ISPs for multiple ISPs in the same region? Is the benefit of SDP obvious for ISPs, CPs, and EUs?

However, these studies have neglected the impact of EUs (*e.g.*, the inherent attribute differences of EUs and the interaction between EUs) on the market. Since SDP can promote more traffic consumption (*e.g.*, more EUs attracted by SDP and more traffic consumption per EU motivated by SDP), it is easy to cause network congestion with the fixed link capacity, illustrated in Fig. 1. Thus, the traffic consumption of one EU affects that of other EUs, which in turn encourages interaction among EUs. Therefore, to fully understand the mechanism of SDP, we must focus on the impact of EUs on the market.

Due to the challenge of modeling the abstract property about interaction among EUs, we utilize network congestion as the medium to describe the interaction among EUs. Moreover, network congestion in reality is a key factor which has a great impact on the traffic consumption of the mobile Internet market [8], [9]. With the continuous improvement of people's living standards, what people care about is not only the price of mobile traffic, but also the quality of experience (QoE) [10]–[12] brought by mobile traffic. Network congestion not only reduces EUs experience, but also affects the revenues of CPs and ISPs. When the network congestion reaches a certain degree, it can give EUs strong negative impacts. Some EUs will even choose to replace their own ISP, resulting in a substantial decline in the revenue of the related ISP. EUs may also think that the technology of associated CPs is defective, and switch to the services of other CPs.

For in-depth understanding the impact of EUs on the market under SDP, in this paper, we focus on the impact of network congestion on SDP. As illustrated in Fig. 1, SDP increases the link utilization rate, which can be regarded as an indication of network congestion. Taking into account that network congestion directly affects the QoE of EUs, we model the impact of network congestion on SDP in the utility function of each EU, further affecting other stakeholders in the market (*i.e.*, ISPs and CPs).

Based on the Stackelberg game model, which is often adopted in existing studies [3], [13]–[15], we first propose a novel congestion-aware SDP model, to describe the relationship among various stakeholders in the market and the revenue of each stakeholder. More specifically, network congestion will bring negative revenue to EUs in our model, and the newly proposed congestion-aware SDP model takes the quadratic function of the link utilization rate to describe the negative impact of network congestion. Like other studies [13], this configuration can achieve the goal that when the degree of network congestion is relatively large, the negative impact is obvious. While the negative impact is negligible with relatively small degree of the congestion. More details can be found in Section III-C.

After constructing the congestion-aware SDP model, it has been demonstrated through experiments that SDP mechanism can indeed increase traffic consumption, thereby increasing the degree of network congestion. This further implies that SDP mechanism is closely related to network congestion. Meanwhile, ISPs, CPs, and EUs all have the characteristics of individual rationality. In other words, they all want to find ways to maximize their revenues based on the characteristics of the market. Based on the newly proposed model, we give out the optimal strategy for each stakeholder in the mobile Internet market under SDPs with considering the impact of network congestion (*i.e.*, the interaction among EUs).

To the best of our knowledge, this paper is the first work which directly analyzes the impact of network congestion (*i.e.*, the interaction among EUs) on SDP from EU perspective, thereby affecting all SDP-related stakeholders in the mobile Internet market. Our main contributions in this paper are summarized as follows:

- Regarding the abstract property about interaction among EUs, our proposed congestion-aware SDP model utilizes network congestion as the medium to analyze the impact of EUs on the market under SDP.
- The congestion-aware SDP model for the first time takes into account the negative impact of network congestion from EU perspective, which can more accurately and realistically describe the optimal strategy of each stakeholder.
- Through the simulation experiments, we have demonstrated that the positive effect that SDPs bring to the market is not as obvious as before, and EUs are no longer independent of each other, but affected by each other.
- Through theoretical modeling and analysis, we prove that there is a unique dynamic equilibrium between the traffic consumption of different EUs, and provide the relevant calculation method.
- Combining theoretical analysis and simulation experiments, we discuss the optimal strategies of various stakeholders, and further emphasizes the non-negligible influence of EUs in the mobile Internet market.

The remainder of this paper is organized as follows. Section II reviews the related work. The details of congestion-aware SDP model are introduced in Section III. In Section IV, we propose the method to get the optimal strategy for each stakeholder in the SDP market. Finally, Section V concludes of this paper.

II. Related Work

As the relationship between supply and demand changes, economics has become an important tool for Internet. To make better use of network resource, various network traffic data pricing schemes are also developing continuously, such as time-dependent pricing [16], connection-based pricing for IoT devices [17], auction-based WiFi pricing [18], *etc.*

SDP [5], [19]–[21], which is the focus of this paper, is a representative solution for smart data pricing. Since SDP allows CPs to pay a percentage of traffic for EUs, the earlier studies [3], [22], [23] on SDP are mainly about the relationship among ISPs, CPs, and EUs. They demonstrate that SDP can create a tripartite win-win phenomenon. With the continuous deepening of research, the inherent attributes or internal competition of a certain type of stakeholders are gradually disclosed. Therefore, relevant research progress and conclusions can be classified from the oriented stakeholders (*i.e.*, ISP, CP, and EU).

ISP-oriented Modeling and Analysis: As the dominant stakeholder in the market, any decision (*e.g.*, link capacity is related to QoS of CPs, traffic price is related to traffic consumption of EUs) made by the ISP affects all stakeholders. Zhang *et al.* [6] find that when the ISP provides sufficient link capacity, SDP will benefit both CPs and EUs in the short-term market. However, in the long-term market, the ISP have no incentive to further improve their services. In addition, when the link capacity is insufficient, SDP enables the ISP and EUs to achieve a win-win phenomenon, but it also leads to the competition between the ISP and CPs for profits. Vyavahare *et al.* [7] analyze how competition among different ISPs affects SDP mechanism. And Vyavahare *et al.* find that SDP can strengthen the dominance of ISPs in the market. Moreover, the competition among ISPs will not diminish this dominance. Overall, the ISP's decision and SDP interact with each other.

CP-oriented Modeling and Analysis: CPs play the role of monetary resource providers for SDP, interacting directly with EUs and ISPs. Accordingly, the decision of CPs can have a direct impact on SDP. For example, CPs expanding the subsidy ratio will incentivize EUs to consume more traffic. Moreover, the services of CPs are diversified, and different CPs have obvious differences in profitability. Thus, the competition between CPs is closely related to SDP [6], [24]. For example, Zhang *et al.* [6] find that SDP will exacerbate the advantage gap between CPs, and CPs with high profitability will have a stronger willingness to support SDP. Similarly, Joe-Wong *et al.* [25] find that SDP is more beneficial to CPs with strong profitability and insensitive expenses. While studying the competition among ISPs, Vyavahare *et al.* [7] also find that when the market deploys the SDP mechanism, ISPs will prefer more profitable CPs.

EU-oriented Modeling and Analysis: Most literatures on SDP assume that the traffic price (or the subsidy ratio) is the core factor affecting the decision-making of EUs. In fact, EUs are the most complex subjects in the market, and their decisions can be affected by a variety of subjective and objective factors. For example, Zhao *et al.* [15], [26] focus on how the intrinsic demand for various contents affects EU decision-making and the market under SDP. Similarly, Joe-Wong *et al.* [25] also further analyze the impact of SDP on EUs. For a given pair of EUs and CPs, Joe-Wong *et al.* [25] find that the increase in EU revenue due to SDP will be more obvious. Moreover, for cost-sensitive EUs, the increase in revenue caused by SDP will be more obvious.

Although the above studies illustrates that EUs can have a significant impact on SDP, the existing studies are still limited to the perspective of individual EUs, such as intrinsic attribute differences. In fact, the interaction among EUs also have an impact on the market under SDP, deserving more in-depth study. Therefore, in this paper, we utilize network congestion as the medium to elucidate the impact of the interaction among EUs on all stakeholders in the market under SDP. Moreover, with the newly proposed congestion-aware SDP model, we not only demonstrate some existing conclusions, but also give some new and revised ones.

III. Congestion-Aware SDP Model

Based on the Stackelberg game model, we propose the congestion-aware SDP model to describe the market under SDPs, which takes into account the impact of network congestion. More specifically, we first model the market under SDP, including the utility functions of all stakeholders, when network congestion is considered. And then, we analyze the necessity of considering network congestion, and determine the form of network congestion in the model. For the sake of clarity, Table I lists the important notations used in this paper. It is worth noting that the specific meaning of the symbol also depends on its superscript and subscript. For example, the traffic consumption of U_i is denoted by x_i, while the optimal traffic consumption of U_i is denoted by ${x_i}^*$. And X represents the total traffic consumption, while X^{-i} represents the total traffic consumption without x_i.

A. Modeling the Market under SDP with Network Congestion

Regarding SDP, the decision-making process of the market conforms to the structural characteristics of *dominant-dominated*. More specifically, ISPs are the dominant players in the market, while CPs and EUs are dominated. Due to this structure, the two-stage Stackelberg game is a common framework for analyzing SDP [6], [15], [26], [27]. Moreover, the ISP, CPs and EUs all want to maximize their own revenue. To achieve this goal, they have their own optimal strategies. In what follows, we briefly describe the makeup of the market under SDP. Immediately after, we elaborate the modeling details in terms of the ISP, CPs, and EUs, respectively.

In terms of modeling and analyzing the market under SDP, it is critical to obtain the optimal traffic consumption of EUs, the optimal traffic subsidy ratio of CPs, and the optimal price of the ISP. In this paper, our focus is the interaction among EUs (*i.e.*, the phenomenon reflected in the medium of network congestion), rather than the competition between ISPs. Therefore, we assume there is only

TABLE I
LIST OF NOTATIONS

Role	Symbol	Meaning
ISP	β_p	CP-oriented price per unit traffic
	β_b	Basic network service fee
	β_u	EU-oriented price per unit traffic
	Δ	Link capacity provided by ISP
	h	Cost per unit link capacity for ISP
	η	Link utilization rate
CPs	v_l	Revenue for P_l with per unit traffic
	α_l	Subsidy ratio provided by P_l
	M	Number of all CPs
EUs	a_i	Intrinsic demand of U_i
	b_i	Demand elasticity of U_i
	ω_i	Sensitivity of U_i to network congestion
	x_i	Traffic consumption of U_i
	X	Total traffic consumption of all EUs
	v_b	Basic revenue of U_i
	γ	Congestion sensitivity level
	N	Number of all EUs

one monopolistic ISP[1]. And a set of CPs are represented by $\mathcal{P} = \{P_1, P_2, \cdots, P_l, \cdots, P_{M-1}, P_M\}$, where a specific CP with index l is $P_l \in \mathcal{P}$, and $M = |\mathcal{P}|$. We assume that there are a set of EUs represented by $\mathcal{U} = \{U_1, U_2, \cdots, U_i, \cdots, U_{N-1}, U_N\}$, where a specific EU with index i is $U_i \in \mathcal{U}$, and $N = |\mathcal{U}|$. The traffic consumed by U_i in a certain billing period[2] is denoted by $x_i \in [0, \infty)$. The traffic consumption of all EUs constitutes a set $\mathcal{X} = \{x_1, x_2, \cdots, x_i, \cdots, x_{N-1}, x_N\}$. The total traffic consumption of all EUs is denoted by $X = \sum_{i \in [1,N]} x_i$, and $X^{-i} = \sum_{j \in [1,N], j \neq i} x_j = X - x_i$ represents the total traffic consumption except for U_i.

Due to the abstract property about interaction among EUs, the related modeling requires network congestion as a medium. For the short-term market, the link capacity that the ISP can provide is fixed, denoted by Δ. Since the link capacity is a limited resource, EUs will influence each other due to network congestion, and then there will be interaction among EUs. However, it is difficult for the traffic consumption to visually reflect network congestion, so we define η to represent the link utilization rate. More specifically, η refers to the total traffic consumption of all EUs divided by the link capacity provided by the ISP, as illustrated in Eq. (1). Obviously, when η exceeds a certain threshold, the degree of network congestion increases with the increase of η.

$$\eta = \frac{X}{\Delta} = \frac{\sum_{i=1}^{N} x_i}{\Delta} = \frac{x_i + X^{-i}}{\Delta} \tag{1}$$

Regarding the relationship between CPs and EUs, each P_l can provide services to multiple EUs at the same time, but we assume that each U_i only enjoys a service provided by a particular P_l. And the mapping between the index of EU and the index of CP can be obtained by Eq. (2). If U_i enjoys multiple different services in reality, we can regard it as multiple virtual EUs. In this case, it has no effect on the considerations of network congestion in SDP, such that the assumption is reasonable.

$$l = \mathcal{L}(i) \tag{2}$$

We model the interaction between the ISP with CPs and EUs as a two-stage Stackelberg game. During the first stage, the monopolistic ISP charges P_l and U_i for consuming traffic, with the price expressed in terms of β_p and β_u, respectively. Since the traffic consumption of a single CP is much larger than that of a single EU, each CP is an important customer of the ISP. Therefore, the CP-oriented price per unit traffic is significantly lower than EU-oriented price per unit traffic. To reflect this, we assume that β_p is less than β_u. Meanwhile, in addition to paying according to traffic consumption, each U_i also pays a fixed basic network service fee β_b, while CPs are not associated with any basic fee. Overall, the revenue of ISP mainly comes from the basic network service fee of all EUs, as well as the traffic consumption costs of all CPs and EUs. Expenditures of ISP mainly include infrastructure costs and daily operating costs. Thus, the utility function of the ISP can be expressed by Eq. (3).

$$R_{ISP}(\beta_u, \beta_p, \beta_b; \theta_{ISP}) = \beta_b \cdot N + (\beta_u + \beta_p) \cdot X - h \cdot \Delta \tag{3}$$

where the fixed attribute parameter set[3] is $\theta_{ISP} = [h]$ and $h \geq 0$. $\beta_b \cdot N$ refers to the basic network service fee of all EUs. Although CPs pay part of the traffic fee for EUs, when calculating the revenue of ISP, it can still be regarded as the ISP charges EUs and CPs according to the traffic consumption, *i.e.*, $(\beta_u + \beta_p) \cdot X$. Δ and h represents the link capacity, and the cost per unit link capacity, respectively. Therefore, $h \cdot \Delta$ represents the infrastructure and daily operating costs.

The second stage is mainly composed of R_{P_l} (*i.e.*, the utility function of P_l) and R_{U_i} (*i.e.*, the utility function of U_i). For each P_l, its utility function can be expressed by Eq. (4).

$$R_{P_l}(\alpha_l; \theta_P^l) = (v_l - \beta_p - \beta_u \cdot \alpha_l) X_l \tag{4}$$

where $\theta_P^l = [v_l]$ and $v_l \geq 0$. α_l is the subsidy ratio of P_l. In the market, different CPs have different profit models. For example, the revenue of Google mainly comes from

[1]We set up this model with one monopolistic ISP not only for mathematical simplicity, but also capture one ISP's monopoly access power for a majority of CPs. Moreover, there are a lot of regions monopolized by one ISP to provide mobile network services, and current long-term contracts also limit EUs' transition from one ISP to another. It is common in reality, like other studies [6], [15], [16], [28].

[2]In this paper, traffic consumption and all stakeholder revenues are calculated within a certain billing period.

[3]Elements in the fixed attribute parameter set are associated with optimal strategy of the ISP. However, these elements are treated as static variables during the model solving process. Similarly, the fixed attribute parameter sets of P_l and U_i are denoted by θ_P^l and θ_U^i, respectively.

advertising. The traffic consumption is associated with the number of advertisement clicks, which indirectly reflects the potential revenue that can be earned. But Amazon is different from Google, and the traffic consumption directly reflects the number of EUs to browse the number of goods. The revenue per unit traffic of Amazon can be higher than that of Google. Therefore, the revenue per unit traffic for different CPs can be different, and can also be the same. With the same reason, the subsidy ratio provided by different CPs can also be different. Thus, in Eq. (4), each P_l has its own revenue per unit traffic, *i.e.*, $v_l \in \mathcal{V} = \{v_1, v_2, \cdots, v_M\}$, and its own subsidy ratio $\alpha_l \in \{\alpha_1, \alpha_2, \cdots, \alpha_M\}$. β_p and $\beta_u \cdot \alpha_l$ respectively refer to the cost incurred by P_l for its own unit traffic consumption and the unit traffic consumption cost caused by providing subsidies for its EUs. Therefore, $v_l - \beta_p - \beta_u \cdot \alpha_l$ refers to the pure revenue that P_l can get from unit traffic consumption. X_l refers to the total traffic consumption related to P_l in a single billing period, and can be calculated by Eq. (5).

$$\begin{aligned} X_l = \sum_{i=1}^{N} x_i' \\ s.t.\ if\ l == \mathcal{L}(i),\ x_i' = x_i;\ otherwise,\ x_i' = 0 \end{aligned} \tag{5}$$

It can be found from Eq. (5), $X_l \leq X$. This is because not all EUs will choose to enjoy the service provided by P_l.

Regarding the utility function about R_{U_i} (the utility function of U_i), it can be expressed by Eq. (6).

$$R_{U_i}(x_i; \theta_U^i) = a_i x_i - b_i {x_i}^2 - f_i(\eta) - (1-\alpha_{\mathcal{L}(i)}) x_i \cdot \beta_u + v_b - \beta_b \tag{6}$$

where $\theta_U^i = [a_i, b_i]$, $a_i > 0$, and $b_i > 0$. $a_i x_i - b_i {x_i}^2 + v_b$ refers to the positive revenue generated via U_i using network services. Specifically, a_i represents the intrinsic demand of U_i, and a greater value of a_i means the larger demand of U_i. b_i represents the demand elasticity. Different from a_i and b_i, which can describe the intrinsic attributes of each U_i, $v_b \geq 0$ is a constant irrelevant to all EUs, which can describe the basis that a EU can obtain within a billing period, and is independent of traffic consumption.

In addition to the positive revenue, U_i also pays ISP for traffic consumption. At the same time, the subsidy ratio provided by CPs to U_i is $\alpha_{\mathcal{L}(i)}$. Therefore, $(1 - \alpha_{\mathcal{L}(i)}) x_i \cdot \beta_u$ in Eq. (6) refers to the traffic consumption fee paid by U_i according to the usage. Meanwhile, the fixed basic network service fee β_b is also part of U_i's expense.

While describing positive revenue and expenses, we utilize $f_i(\eta)$ in Eq. (6) to describe the negative benefits that network congestion brings to U_i. Combining the Eq. (1), it can be found that the negative benefits caused by network congestion are related to the total traffic consumption X, so EUs interact with each other. Moreover, each EU has a different sensitivity to network congestion, so f_i is associated with the index of U_i. More details about $f_i(\eta)$ can be found in Section III-C, in which the utility function with quadratic form are widely adopted to analyze the impact of congestion on EUs [13].

B. Necessity of Considering Network Congestion

In the Stackelberg game model for SDP, the first step is to compute the optimal strategies for CPs and EUs in the second stage with assuming that the price (*i.e.*, β_p, β_u, β_b) is decided by ISP and the available link capacity Δ is known. The optimal strategies refer to the best subsidy ratio α^* of CPs and the optimal traffic consumption $\mathcal{X}^*$ of EUs. To illustrate the necessity of considering network congestion, we first assume that the utility function of each EU is independent of network congestion. Thus, Eq. (6) can be simplified as Eq. (7).

$$R_{U_i}(a_i, b_i, x_i) = a_i x_i - b_i {x_i}^2 - (1-\alpha_{\mathcal{L}(i)}) x_i \cdot \beta_u + v_b - \beta_b \tag{7}$$

Considering that each U_i wants to maximize his or her own revenue, we have first-order derivative $\frac{\partial R_{U_i}(x_i; \theta_U^i)}{\partial x_i} = 0$ to find the best traffic consumption for each U_i. Specifically, the optimal traffic consumption of each U_i can be expressed by ${x_i}^*$, which is illustrated as Eq. (8).

$${x_i}^* = \frac{a_i - (1 - \alpha_{\mathcal{L}(i)})\beta_u}{2b_i} \tag{8}$$

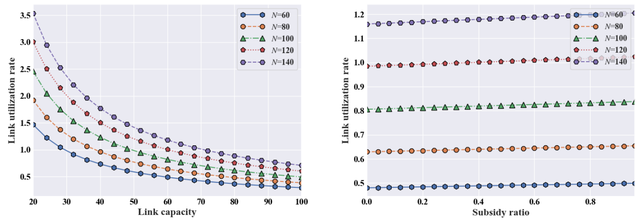
The link utilization rate (*i.e.*, η) defined in Section III-A can indirectly describe the degree of network congestion. After clarifying the optimal traffic consumption ${x_i}^*$, η can be further updated. That is, if the utility function of U_i is independent of network congestion, Eq. (1) will be updated to Eq. (9).

$$\eta = \frac{X^*}{\Delta} = \frac{\sum_{i=1}^{N} {x_i}^*}{\Delta} = \sum_{i=1}^{N} \frac{a_i - (1 - \alpha_{\mathcal{L}(i)})\beta_u}{2b_i \cdot \Delta} \tag{9}$$

From Eq. (9), we can see that, if the utility function is independent of network congestion, both the increase in the number of EUs (*i.e.*, N) and the improvement of subsidy ratio of CPs (*i.e.*, $\alpha_{\mathcal{L}(i)}$) will increase η. Meanwhile, the link utilization rate is closely related to network congestion. If EUs make decisions without considering the impact of network congestion, the introduction of SDP can significantly improve link utilization rate, thereby increasing the probability of network congestion.

To visually illustrate the necessity of considering network congestion, we have performed some simulation experiments, as illustrated in Fig. 2. In our experiments, the attributes of EUs are subject to a Gaussian distribution. For $\{a_i | i \in [1, N]\}$, the mean parameter is 10 and standard deviation parameter is 1. For $\{b_i | i \in [1, N]\}$, the mean parameter is 10 and standard deviation parameter is 1. When analyzing the impact of link capacity, the subsidy ratio is 40%.

Specifically, Fig. 2 illustrates how the number of EUs (*i.e.*, N), the link capacity provided by ISP (*i.e.*, Δ), and the subsidy ratio of CPs (*i.e.*, $\alpha_{\mathcal{L}(i)}$) affect network congestion (*i.e.*, η). In Fig. 2(a), we can see that with the increase in link capacity, η gradually declines, which is consistent with Eq. (9). Due to the huge economic costs, however, it is hard for the ISP to expand link capacity. Especially in the short-term market, ISP cannot instantly increase link capacity. From the Fig. 2(b), it can be found that for a scenario with a fixed



(a) Link utilization rate (*i.e.*, η) *vs.* Link capacity (*i.e.*, Δ)

(b) Link utilization rate (*i.e.*, η) *vs.* Subside ratio (*i.e.*, α)

Fig. 2. Impact of number of EUs (*i.e.*, N), link capacity provided by ISP (Δ), and subsidy ratio of CPs (*i.e.*, α) on link utilization rate (*i.e.*, η).

Δ, as the subside ratio increases, η will continue to gradually increase regardless of whether the link utilization rate exceeds 1. This phenomenon occurs because the negative impact of congestion on traffic consumption is ignored. In other words, EUs cannot perceive changes in the total traffic consumption (*e.g.*, more EUs attracted by the increased α and more traffic consumption per EU motivated by the increased α) through network congestion. In addition, in Fig. 2(a) and Fig. 2(b), as the numebr of EUs increases, η continues to increase, and the probability of network congestion also increases.

Overall, once Eq. (7) is independent of network congestion, then EUs are also independent of each other. Moreover, SDP not only attracts more EUs to enter the market, but also motivates each EU to consume more traffic [6], [15]. These factors will further improve the link utilization rate in the short-term market. Therefore, when modeling the utility function of U_i, network congestion must be considered, as well as the interaction among EUs. Note that Eq. (7), Eq. (8) and Eq. (9) in this section are only used to analyze the necessity of considering network congestion. In the following content, we will utilize the congestion-aware model (*i.e.*, the relevant model in Section III-A), to re-solve the optimal strategies and perform relevant analysis.

C. Network Congestion Modeling

It can be found that the increase in α of CPs can significantly increase the traffic consumption of EUs. Without considering network congestion, EUs will consume traffic consumption the ideal optimal traffic consumption, illustrated in Eq. (8). Moreover, the total traffic consumption will increase linearly with the increase of N. In reality, the number of EUs for the same ISP is huge, which will inevitably lead to network congestion. More seriously, SDP will aggravate network congestion. However, network congestion is closely related to QoS or QoE, which is bound to affect traffic consumption. Therefore, in this part, we model network congestion and determine the form of network congestion in the congestion-aware SDP model.

Network congestion is caused by the total traffic consumption exceeding the link capacity, and these traffic consumption can come from different EUs. Therefore, once network congestion is considered, the optimal strategy of U_i is no longer a matter of its own, but is also influenced by other EUs decisions.

In this paper, we apply link utilization rate (*i.e.*, η) to indirectly reflect network congestion. η is related to the traffic consumed by each U_i in the market, and using η to indirectly reflect network congestion can establish connections between EUs, which facilitates the analysis of the interaction among EUs. As illustrated in Eq. (6), network congestion will reduce the revenue of EUs, that is, $f_i(\eta) > 0$. Moreover, when η is very low, the negative benefits brought by network congestion should be tiny. And when η is relatively high (*e.g.*, close to 1 or even larger than 1), the negative benefits brought by network congestion will increase significantly. To meet these characteristics, we apply the quadratic function [13] of η, rather than the linear function of η, to define $f_i(\eta)$. At the same time, the negative benefits of network congestion is related to the degree of network congestion as well as the attributes of EUs (*e.g.*, sensitivity to network congestion). Therefore, we define $\omega_i \in [0, +\infty)$ to indicate the sensitivity of U_i to network congestion.

For the above reasons, we describe the network congestion via link capacity of the ISP and total traffic consumption of all EUs, as illustrated in Eq. (10).

$$f_i(\eta) = \omega_i(\frac{X}{\Delta})^2 = \omega_i(\frac{x_i + X^{-i}}{\Delta})^2 = \omega_i(\frac{x_i + \sum\limits_{j\in[1,N], j\neq i} x_j}{\Delta})^2 \tag{10}$$

IV. Optimal Strategy Derivation and Analysis

Based on the newly proposed congestion-aware SDP model, we solve the model to get the optimal strategy of each stakeholder in the mobile Internet market under SDPs.

A. Optimal Strategy of EUs

For each EU, the attributes of U_i and the service enjoyed by U_i (*i.e.*, the associated $P_{\mathcal{L}(i)}$) is determined. In other words, a_i, b_i, ω_i, and $\alpha_{\mathcal{L}(i)}$ are all known when we compute the optimal strategy for U_i. The optimal strategy refers to how much traffic U_i should consume to maximize his or her own revenue R_{U_i}. By substituting Eq. (10) into Eq. (6), it can be found that R_{U_i} is the quadratic function of x_i. Thus, through the first-order derivative $\frac{\partial R_{U_i}(x_i;\theta_U^i)}{\partial x_i} = 0$, we can get the optimal traffic consumption for U_i, which is illustrated as Eq. (11).

$$x_i^* = \frac{a_i - (1-\alpha_{\mathcal{L}(i)})\beta_u - \frac{2\omega_i}{\Delta^2}\sum\limits_{j\in[1,N], j\neq i} x_j^*}{2b_i + \frac{2\omega_i}{\Delta^2}} \tag{11}$$

Regarding Eq. (11), when η is relatively low (*i.e.*, the link capacity Δ is relatively large), the interaction among EUs is very slight. In this case, there is no congestion in the network, and the traffic consumption of each U_i only depends on the traffic price and their demands. Comparing Eq. (11) and Eq. (8), it can be found that when $\Delta \to +\infty$, Eq. (11) is equivalent to Eq. (8). However, the infinite link capacity is impossible. The larger link capacity means the higher infrastructure cost of ISP. Typically, the ISP only provides the link capacity close to the market demand, or even slightly less than the market demand. At this point, the phenomenon of network congestion will occur, and each EU's decision is influenced by traffic consumption of other EUs. As a result, EUs will interact with each other. And each U_i wants to maximize his or her own revenue, so as to achieve a dynamic equilibrium. The traffic consumed by each EU in the dynamic equilibrium is the optimal strategy for each U_i. In other words, the optimal traffic consumption of all EUs constitutes a game.

In this paper, the optimal traffic consumption game for all EUs is defined as the user demand game, denoted by $\mathcal{G} \triangleq \{\mathcal{U} = \{U_1, U_2, \cdots, U_i, \cdots, U_{N-1}, U_N\}, R_U = \{R_{U_i}(x_i; \theta_U^i)|i \in [1, N]\}, \mathcal{X} = \{x_i|i \in [1, N], x_i \in [0, +\infty)\}\}$. Specifically, $\mathcal{U}$, R_U, and $\mathcal{X}$ refer to all EUs participating in the game, the utility functions of all EUs, and the policy space of all EUs, respectively. To prove that there is a dynamic equilibrium in the user demand game, and it is the only dynamic equilibrium, we have Theorem 2 and Theorem 4. Before proving Theorem 2, we first give Lemma 1.

Lemma 1: The user demand game $\mathcal{G}$ has the same dynamic equilibrium as $\mathcal{G}'$, where $\mathcal{G}' \triangleq \{\mathcal{U} = \{U_1, U_2, \cdots, U_i, \cdots, U_{N-1}, U_N\}, R_U = \{R_{U_i}(x_i; \theta_U^i)|i \in [1, N]\}, \mathcal{X} = \{x_i|i \in [1, N], x_i \in [0, \bar{x})\}\}$. And $\bar{x}$ can be any number satisfying Eq. (12).

$$\bar{x} > \max_{i \in [0,N]} \frac{|a_i - (1 - \alpha_{\mathcal{L}(i)})\beta_u|}{2b_i + \frac{2\omega_i}{\Delta^2}} \tag{12}$$

Proof of Lemma 1: Assume $\mathcal{X}^*$ is the optimal traffic consumption set of all EUs when the user demand game $\mathcal{G}$ reaches dynamic equilibrium, and x_i^* is the maximum value in $\mathcal{X}^*$, *i.e.*, $x_i^* \geq x_j^*, \forall i \neq j$.

If $x_i^* = 0$, since x_i^* is the maximum value in $\mathcal{X}^*$, and $x_i^* \geq x_j^*, \forall i \neq j$, it can be known that the traffic consumption of all EUs is 0.

If $x_i^* > 0$, according to Eq. (11), we have Eq. (13).

$$\begin{aligned} x_i^* &= \frac{a_i - (1-\alpha_{\mathcal{L}(i)})\beta_u}{2b_i + \frac{2\omega_i}{\Delta^2}} - \frac{\frac{\omega_i}{\Delta^2} \sum_{j \in [1,N], j \neq i} x_j^*}{b_i + \frac{\omega_i}{\Delta^2}} \\ &\leq \frac{|a_i - (1-\alpha_{\mathcal{L}(i)})\beta_u|}{2b_i + \frac{2\omega_i}{\Delta^2}} - \frac{\frac{\omega_i}{\Delta^2} \sum_{j \in [1,N], j \neq i} x_j^*}{b_i + \frac{\omega_i}{\Delta^2}} \\ &\leq \frac{|a_i - (1-\alpha_{\mathcal{L}(i)})\beta_u|}{2b_i + \frac{2\omega_i}{\Delta^2}} \end{aligned} \tag{13}$$

Combining Eq. (12) and Eq. (13), we can get Eq. (14).

$$x_i^* \leq \frac{|a_i - (1 - \alpha_{\mathcal{L}(i)})\beta_u|}{2b_i + \frac{2\omega_i}{\Delta^2}} \leq \bar{x} \tag{14}$$

Since x_i^* is the maximum value in $\mathcal{X}^*$, we have Eq. (15).

$$x_i^* \leq \bar{x}, \forall i \in [1, N] \tag{15}$$

Therefore, the user demand game $\mathcal{G}$ has the same strategy space as the $\mathcal{G}'$, *i.e.*, the same dynamic equilibrium. That is, Lemma 1 is proven. ∎

Theorem 2 (Existence of Dynamic Equilibrium): For the user demand game $\mathcal{G}$, a dynamic equilibrium exists.

Proof of Theorem 2: According to the conclusions already proved in the existing literatures [29]–[31], for an infinite game $\mathcal{G}'$, if the utility function of each EU participating in the game is a continuous concave function, and the policy space of all EUs $\mathcal{X} = \{x_i|i \in [1, N], x_i \in [0, \bar{x})\}$ is both convex and compact, and then there exists a dynamic equilibrium in $\mathcal{G}'$.

According to the definition of $\mathcal{G}'$, it can be found that the above conditions are satisfied, so there is a dynamic equilibrium in $\mathcal{G}'$. According to Lemma 1, therefore, a dynamic equilibrium exists in the user demand game $\mathcal{G}$. ∎

To simplify the proof of Theorem 4, we have Assumption 3.

Assumption 3: For each U_i, it has the characteristics illustrated by Eq. (16).

$$b_i > \frac{\omega_i(N-2)}{\Delta^2}, \quad \forall i \tag{16}$$

Theorem 4 (Uniqueness of Dynamic Equilibrium): For the user demand game $\mathcal{G}$, the dynamic equilibrium is unique.

Proof of Theorem 4: We define the utility function configuration of all EUs in the game $\mathcal{G}'$ as $R_U(\mathcal{X}) \triangleq (R_{U_1}(\mathcal{X}), R_{U_2}(\mathcal{X}), \cdots, R_{U_N}(\mathcal{X}))$, and the corresponding Hessian matrix is denoted by $\nabla R_U(\mathcal{X})$. More details about $\nabla R_U(\mathcal{X})$ can be found in Eq. (17).

$$\begin{aligned} \nabla R_U(\mathcal{X}) &= \begin{bmatrix} \frac{\partial^2 R_{U_1}(\mathcal{X})}{\partial x_1^2} & \frac{\partial^2 R_{U_1}(\mathcal{X})}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 R_{U_1}(\mathcal{X})}{\partial x_1 \partial x_N} \\ \frac{\partial^2 R_{U_2}(\mathcal{X})}{\partial x_2 \partial x_1} & \frac{\partial^2 R_{U_2}(\mathcal{X})}{\partial x_2^2} & \cdots & \frac{\partial^2 R_{U_2}(\mathcal{X})}{\partial x_2 \partial x_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 R_{U_N}(\mathcal{X})}{\partial x_N \partial x_1} & \frac{\partial^2 R_{U_N}(\mathcal{X})}{\partial x_N \partial x_2} & \cdots & \frac{\partial^2 R_{U_N}(\mathcal{X})}{\partial x_N^2} \end{bmatrix} \\ &= \begin{bmatrix} -2b_1 - \frac{2\omega_1}{\Delta^2} & -\frac{2\omega_1}{\Delta^2} & \cdots & -\frac{2\omega_1}{\Delta^2} \\ -\frac{2\omega_2}{\Delta^2} & -2b_2 - \frac{2\omega_2}{\Delta^2} & \cdots & -\frac{2\omega_2}{\Delta^2} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{2\omega_N}{\Delta^2} & -\frac{2\omega_N}{\Delta^2} & \cdots & -2b_N - \frac{2\omega_N}{\Delta^2} \end{bmatrix} \\ &= -H \end{aligned} \tag{17}$$

where H is defined by Eq. (18).

$$H = \begin{bmatrix} 2b_1 + \frac{2\omega_1}{\Delta^2} & \frac{2\omega_1}{\Delta^2} & \cdots & \frac{2\omega_1}{\Delta^2} \\ \frac{2\omega_2}{\Delta^2} & 2b_2 + \frac{2\omega_2}{\Delta^2} & \cdots & \frac{2\omega_2}{\Delta^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{2\omega_N}{\Delta^2} & \frac{2\omega_N}{\Delta^2} & \cdots & 2b_N + \frac{2\omega_N}{\Delta^2} \end{bmatrix} \tag{18}$$

According to Assumption 3, we have Eq. (19).

$$H_{i,i} > \sum_{j \in [1,N], j \neq i} H_{i,j}, \quad \forall i \tag{19}$$

Hence, H is strictly a diagonal dominance matrix [32]. And then, we have $H + H^T$ is a symmetric matrix with strictly diagonal dominance.

According to the existing literature [13], [32], a symmetric matrix with diagonal dominance and non-negative diagonal elements, the matrix is positive definite. Therefore, $H + H^T$ is a positive definite matrix. Correspondingly, $\nabla R_U(\mathcal{X}) + \nabla R_U(\mathcal{X})^T = -H - H^T$ is a negative definite matrix. According to Theorem 6 in the literature [33], $R_U(\mathcal{X})$ is diagonally strictly concave. According to Theorem 2 in the literature [33], the dynamic equilibrium of the game $\mathcal{G}'$ has unique properties. Finally, combined with the Lemma 1, the dynamic equilibrium of the user demand game $\mathcal{G}$ has unique properties.

■

According to Theorem 2 and Theorem 4, we can find that, for each U_i in the market, there is an unique value with Eq. (11). Therefore, we can provide the optimal strategy for each U_i. Specifically, we can utilize **Algorithm 1** to calculate the dynamic equilibrium of traffic consumption.

Algorithm 1: Optimal traffic consumption of all EUs

$k \leftarrow 0$, $flag \leftarrow 1$
while $flag > 0$ **do**
  $flag \leftarrow 0$
  foreach $U_i \in \mathcal{U}$ **do**
    if k *is equal to* 0 **then**
      Randomly initialize $x_i \in \mathcal{X}$ by $r \in [0, \Delta]$
    else
      Calculate $x_i^{*(k)}$ by Eq. (11) with $\alpha_{\mathcal{L}(i)} \leftarrow 0$
    $x_i^* \leftarrow x_i^{*(k)}$
    if $|x_i^* - x_i^{*(k-1)}| > \tau$ **then**
      $flag \leftarrow 1$
  $k \leftarrow k + 1$
return $\mathcal{X}^*$

Since the decision of EUs is in the second stage of the Stackelberg game, we assume that the traffic price of ISP and the subsidy ratio for CPs are known as input for **Algorithm 1**. The only thing we need to do is to calculate the optimal traffic consumption (*i.e.*, $\mathcal{X}$), including how much traffic to consume for each U_i. After continuous iteration, **Algorithm 1** gradually reaches the equilibrium state among EUs, and the final output is the optimal traffic consumption of each U_i.

B. Optimal Strategy of CPs

Regarding the second stage of Stackelberg game, it involves the optimal strategies of EUs and CPs. Since we have already derived the former, in this part, we will introduce how to get the optimal strategy of CPs.

CPs partially pay traffic consumption for EUs, which can motivate EUs to consume more traffic. And then, this can improve CPs' own revenue through commercial activities such as advertising. To clarify the relationship between the revenue of CPs and the subsidy ratio provided by CPs, we assume that there is only one CP in the market. Therefore, α_l and X_l can be simplified to α and X, respectively. Through substituting Eq. (11) into Eq. (4), we can see that the revenue of P_l is the quadratic function of α_l, and the coefficient of the quadratic term is negative. In other words, the curve of R_{P_l} about α_l is a parabola, of which the opening direction is downward. According to the nature of the quadratic function, the revenue of P_l has a unique maximum value.

For Eq. (11), we replace $\sum_{j \in [1,N], j \neq i} x_j^*$ with $X - x_i^*$. And then, we have Eq. (20).

$$x_i^* = \frac{a_i - (1-\alpha)\beta_u - \frac{2\omega_i}{\Delta^2} X}{2b_i} \tag{20}$$

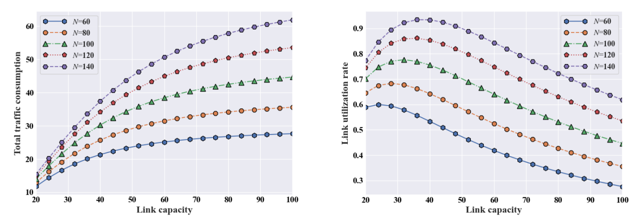
In fact, $\sum_{i \in [1,N]} x_i^* = X$. Combined with Eq. (20), the total traffic consumption of all EUs can be obtained via Eq. (21).

$$X = \frac{\sum_{i \in [1,N]} \frac{a_i - (1-\alpha)\beta_u}{2b_i}}{1 + \frac{1}{\Delta^2} \sum_{i \in [1,N]} \frac{\omega_i}{b_i}} \tag{21}$$

By substituting Eq. (21) into Eq. (4), and solving the first-order derivative $\frac{\partial R_{P_l}(\alpha_l; \theta_P^l)}{\partial \alpha_l} = 0$, we can obtain the optimal subsidy ratio, as illustrated in Eq. (22).

$$\alpha^* = \frac{\sum_{i \in [1,N]} \frac{v + \beta_u - \beta_p - a_i}{2b_i}}{\beta_u \sum_{i \in [1,N]} \frac{1}{b_i}} \tag{22}$$

With Eq. (21) and Eq. (22), we utilize simulation experiments to further clarify that when analyzing the SDP mechanism, network congestion is a factor that cannot be ignored. The relevant simulation results are illustrated in Fig. 3.



(a) Total traffic consumption (*i.e.*, X) *vs.* Link capacity (*i.e.*, Δ)
(b) Link utilization rate (*i.e.*, η) *vs.* Link capacity (*i.e.*, Δ)

Fig. 3. Impact of link capacity (*i.e.*, Δ) on total traffic consumption (*i.e.*, X) and link utilization rate (*i.e.*, η).

Fig. 3 illustrates that under different fixed link capacity scenarios, the total traffic consumption of all EUs and the change in link utilization rate. In terms of experimental configuration, the EU's fixed attributes a_i and b_i are the same as the experimental configuration in Section III-B. Moreover, the congestion sensitivity of all EUs $\{\omega_i | i \in [1, N]\}$ is also selected according to the Gaussian distribution, where the mean parameter is 100, and the standard deviation parameter is 1. When network congestion is not considered, according to Eq. (8), the link capacity cannot affect the traffic consumption. However, when network congestion is considered, link capacity has a significant impact on the traffic consumption and link utilization rate.

In Fig. 3(a), as the link capacity increases, the total traffic consumption of all EUs increases gradually. This is because the increase in link capacity reduces the probability of network congestion, thereby weakening the interaction among EUs, and EUs will increase traffic consumption. Moreover, with the increase in link capacity, the negative impact of network congestion is gradually weakened, and the increase in traffic consumption is gradually slows down. By observing the different curves in Fig. 3(a), it can be found that a larger number of EUs will promote tighter interaction among EUs, and the change in traffic consumption caused by the increase in link capacity will be more obvious.

Different from the phenomenon in Fig. 2(a), Fig. 3(b) illustrates that in some scenarios, the increase in link capacity will further improve link utilization rate. This is because, in the case of low link capacity, increasing link capacity can significantly alleviate the negative impact of network congestion, resulting in an increased link utilization rate. When the link capacity exceeds a certain threshold, the effect of alleviating network congestion due to the increase in link capacity is weakened, and the link utilization rate also decreases. Moreover, whether it is Fig. 3(a) or Fig. 3(b), we can find that the number of EUs affects the impact of link capacity.

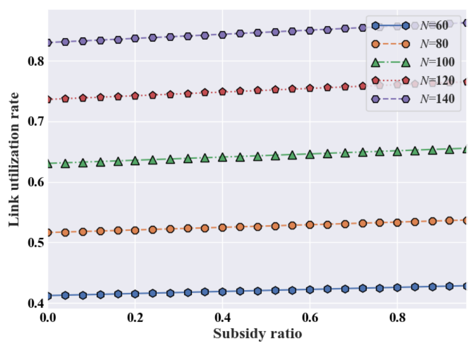


Fig. 4. Impact of subsidy ratio (*i.e.*, α) on link utilization rate (*i.e.*, η).

In addition to link capacity, we further analyze the impact of the subsidy ratio on link utilization rate. In Fig. 2(b), the link utilization rate of some experimental results exceeds 1. In Fig. 4, it can be found that although the increase of subsidy ratio can improve the link utilization rate, the link utilization rate is always lower than 1 due to the interaction among EUs. Moreover, compared to the curves of the same number of EUs in Fig. 2(b), the link utilization rate illustrated by the relevant curves in Fig. 4 is significantly lower. This again demonstrates that the interaction among EUs caused by network congestion cannot be ignored.

C. Optimal Strategy of ISP

We have already completed the discussion about the second stage of the Stackelberg game model in Section IV-A and Section IV-B by analyzing the optimal strategies for EUs and CPs. And in this part, we will analyze its first stage by discussing how the monopolistic ISP maximizes its own revenue through the optimal strategy.

As the dominance of the Stackelberg game model, the monopolistic ISP has an important impact on the market. It also plays a decisive role in deciding whether SDP can be widely used in the market. In terms of ISP, the optimal strategy involves the traffic price and link capacity, *i.e.*, β_b, β_p,, β_u, and Δ. Considering that it takes a lot of time and money to expand link capacity, we assume that the link capacity is a fixed value. And then, the optimal strategy for the monopolistic ISP is to determine the optimal prices, including β_b, β_p, and β_u.

Considering that CPs are large customers of the ISP, the price of traffic for CPs is relatively stable and is a small value. Furthermore, we focus on the interaction among EUs. To simplify the problem, we assume that β_p and β_b are fixed, and only β_u is a variable. Therefore, the fixed attribute parameter set of the ISP is updated to $\theta_{ISP} = [h, \Delta, \beta_p, \beta_b]$. Through simulation experiments, Fig. 5 illustrates how the ISP dominates the market under SDP through traffic price.

In Fig. 5(a), the total traffic consumption of all EUs decreases as the traffic price increases. This is because, even if the CPs associated with EUs offer subsidies, EUs still need pay part of the traffic consumption cost. Under the condition that the EU's intrinsic demand remains unchanged, the increase of the price β_u will lead to an increase in the cost of traffic usage, thereby reducing the traffic consumption of EUs. In addition, we can find that the impact of traffic price becomes more pronounced as the number of EUs increases.

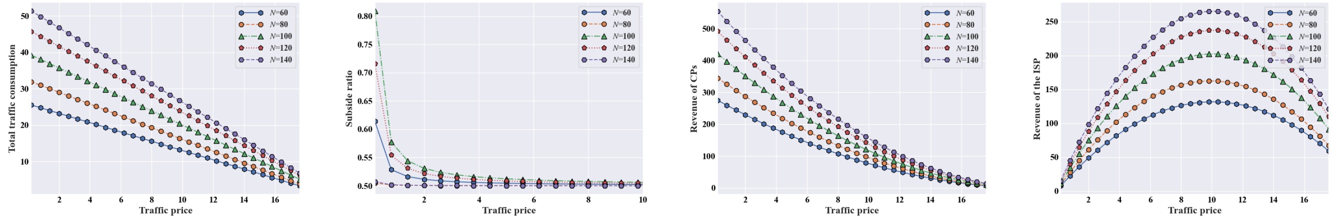
Regarding the impact of ISP decisions on CPs, it can be found in Fig. 5(b) and Fig. 5(c). Although we assume that the traffic price β_p is fixed, CPs still need to pay part of the traffic consumption fee for EUs. Therefore, Fig. 5(b) illustrates that the subsidy ratio α will decrease as the traffic price β_u increases, which is consistent with Eq. (22). Moreover, Fig. 5(b) also illustrates that CPs will reduce the subsidy ratio to cope with the increase in traffic price.

However, with the reduction of traffic consumption, the revenue of CPs in Fig. 5(c) will still decrease with the increase of traffic price. This is because the revenue of CPs mainly depends on the revenue of commercial activities such as advertisements [34] viewed (or clicked) by EUs when consuming traffic, and these commercial activities are closely related to the traffic consumption.

It can be found that in Fig. 5(d), the change of R_{ISP} with respect to the traffic price β_u presents a parabolic shape (*i.e.*, an approximate quadratic function of β_u), indicating that the ISP has a unique optimal strategy in terms of traffic price. Overall, the analysis on Fig. 5 demonstrates that the optimal strategy of ISP has a significant impact on all stakeholders.

By substituting Eq. (21) and Eq. (22) into Eq. (3), it can be found that R_{ISP} is the quadratic function of β_u, and the coefficient of the quadratic term is negative. That is to say, the curve of R_{ISP} about β_u is a parabola, and the opening direction is downward, which is consistent with the experimental results illustrated in Fig. 5(d). According to the nature of the quadratic function, R_{ISP} has a unique maximum value. Thus, with the first-order derivative $\frac{\partial R_{ISP}(\beta_u;\theta_{ISP})}{\partial \beta_u} = 0$, the EU-oriented optimal traffic price β_u^* can be calculated by Eq. (23).

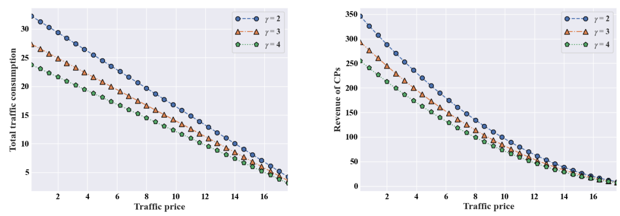
$$\beta_u^* = \frac{\sum\limits_{i\in[1,N]} \frac{v-2\beta_p}{2b_i}}{\sum\limits_{i\in[1,N]} \frac{1}{b_i}} \tag{23}$$



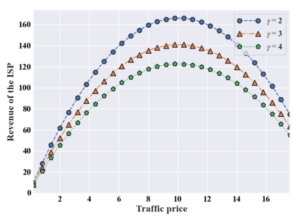
(a) Total traffic consumption (*i.e.*, X) *vs.* Traffic price (*i.e.*, β_u) (b) Subsidy ratio of CPs (*i.e.*, α) *vs.* Traffic price (*i.e.*, β_u) (c) Revenue of CPs (*i.e.*, R_{P_l}) *vs.* Traffic price (*i.e.*, β_u) (d) Revenue of ISP (*i.e.*, R_{ISP}) *vs.* Traffic price (*i.e.*, β_u)

Fig. 5. Analysis of the dominance of the ISP in the market by virtue of traffic price.

To analyze the impact of network congestion sensitivity on the market, we further define the congestion sensitivity level, denoted by γ. On the basis of the above simulation experiments, we take the product of ω_i and γ as the congestion sensitivity in the simulation experiment, that is, $\omega_i' = \omega_i \cdot \gamma, \forall i$.



(a) Total traffic consumption (*i.e.*, X) *vs.* Traffic price (*i.e.*, β_u) (b) Revenue of CPs (*i.e.*, R_{P_l}) *vs.* Traffic price (*i.e.*, β_u)



(c) Revenue of the ISP (*i.e.*, R_{ISP}) *vs.* Traffic price (*i.e.*, β_u)

Fig. 6. Impact of EU congestion sensitivity level on the market.

Through Fig. 6, it can be found that the traffic price set by the ISP still dominates the market, and the impact of the traffic price is qualitatively similar to that illustrated in Fig. 5. The results remain the same. By analyzing the different curves in Fig. 6(a), Fig. 6(b) and Fig. 6(c), it can be found that in terms of quantitative analysis, the congestion sensitivity level γ can affect the dominance of traffic price. To be precise, with the increase of the congestion sensitivity level γ, other factors caused by price changes (*e.g.*, the total traffic consumption of all EUs, the revenue of CPs, and the revenue of ISP) become more stable. This is because a higher congestion sensitivity level γ means that the interaction among EUs is more obvious, which in turn weakens the impact of traffic price on the market under SDP. Overall, the congestion sensitivity level is an intrinsic attribute of EUs, which also demonstrates that the modeling and analysis of SDP mechanism should focus on the influence of EUs.

V. Conclusion

Since EUs are an important part of the mobile Internet market, and the research on EU-oriented SDP mechanism is insufficient, this paper proposes a congestion-aware SDP model. The newly proposed model utilizes network congestion as the medium and emphasizes on the impact of interaction among EUs on the market under SDP, rather than the impact of CPs and ISPs. Specifically, based on the Stackelberg game, we first model the interactions of all stakeholders in the market as well as the utility function. After analyzing the necessity of network congestion, we use link utilization rate to define the specific form of network congestion, so that network congestion affects all stakeholders from EU's level. Based on our proposed model, we present the optimal strategies for all stakeholders and prove that there is a dynamic equilibrium in the interactions among EUs (*i.e.*, the traffic consumption of each EU). Moreover, the experimental results demonstrate that once the interaction among EUs is considered, the incentive effect of SDP on the market will be weakened. This is because the decision-making process of EU is no longer solely dependent on the traffic price, but is also affected by its own attributes and interaction among EUs. Overall, this paper demonstrates some existing conclusions, but also gives some new conclusions closely related to EUs, which can guide the relevant stakeholders to make more realistic decisions.

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