Variety Matters: A New Model for the Wireless Data Market under Sponsored Data Plans

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ABSTRACT

In this paper, we develop a new model to study the competition among Content Providers (CPs) under Sponsored Data Plans (SDPs). SDP is an emerging pricing model for the wireless data market where Internet Service Providers (ISPs) allow a CP to compensate the traffic volume of users when users access the contents of this CP. Studies have shown that SDPs create a triple-win situation, where users consume more contents and the revenue of both CPs and ISPs increases. Currently, a main concern of SDPs is on whether SDPs may bring about unfair competition among CPs. Studies have shown that big CPs have an advantage over small CPs. We observe that such conclusions are derived because in all previous models, traffic price is the only factor that affects user decisions. We argue that it is not precise. Nowadays, people conduct a large variety of activities online, and users have an intrinsic demand for a variety of contents. To reflect this, we for the first time characterize the variety demand as an intrinsic parameter of users, and integrate such variety into a new model to help us drive some novel insights into SDPs, especially the competition among CPs. Our model shows that variety matters for understanding SDPs more thoroughly and comprehensively. For example, under SDPs, the advantage of CPs with higher revenue will be significantly reduced if users have a greater love for variety. Overall, our new model leads to a set of completely new results and rectifies some past conclusions.

CCS CONCEPTS

• **Networks** → **Network economics**; *Network performance modeling*; Public Internet;

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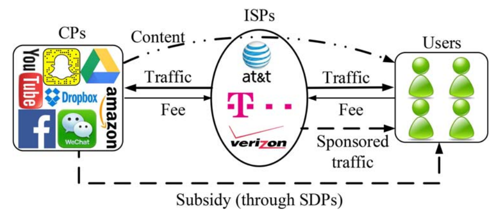

Figure 1: The wireless data market under SDPs.

KEYWORDS

Sponsored Data Plan (SDP), Relative Love for Variety (RLV), Competition Among CPs

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1 INTRODUCTION

With an increasing number of smart devices and applications, mobile data traffic increases tremendously. It poses a great burden to Internet Service Providers (ISPs) since financing such supply-demand gap requires a large investment. Recently, Sponsored Data Plans (SDPs) are proposed and attract interests from both academia and industries. Specifically, SDP means that an ISP and a Content Provider (CP) make an agreement that when users access the contents provided by this CP, users' traffic will be (partially) paid by the CP, as shown in Figure 1. For a new pricing model, it is important to understand what SDPs will bring about to the market, so that players in the market and regulatory bodies can take appropriate actions. Early studies develop models to study the interactions among users, ISPs and CPs, and confirm that SDPs create a triple-win situation[1, 12]. Recent studies develop models to analyze the impact of SDPs on the competition among different CPs [21, 22]. Conclusions of these models include that SDPs may benefit big CPs who can afford a higher sponsored level, which bring about serious concerns on SDPs.

We observe that in all these models, traffic price (e.g., sponsorship) is the only factor that affects user decision optimizations. As a consequence, a CP can easily drive user decisions by a slight increase in the sponsored level. We argue that these models simplify an important factor. Nowadays, people have become accustomed to a wide variety of services over the Internet, e.g., e-shopping, online meetings, and online entertainments. This can be reflected in many practical applications and research areas. For example, considering the frequent use of multiple services, recommendation system can use behavioral data from multiple sources to infer friendship [23]. For people, there is an intrinsic demand for a variety of contents (we also call it variety demand). Such demand cannot be easily substituted by sponsored traffic. Models overlooking this factor may give imprecise or even wrong conclusions. Therefore, based on access time, we develop a new model to characterizes this factor and reconsider what SDPs will bring about to the market, especially the competition among CPs.

There are many challenges. First, we need to capture the variety demand. Variety demand cannot simply be characterized as a cost of user. The nature of the variety demand is how hard it is to replace many different contents with a single content. In other words, if the variety demand is high, a user want to access more different contents during a fixed period. This is independent of cost, and we need a new index. Second, it is also challenging to integrate such new index into the overall model of the market with SDPs. Third, for a real market, the variety demand is abstract and its value may not be directly available. In this paper, we characterize variety demand by using the notion of substitution and define a new index called Relative Love for Variety (RLV). We integrate RLV into an overall two-stage Stackelberg game model after a series of transformations. We comprehensively analyze our new model and obtain many results:

- **The influence of SDPs is not as significant as we believed before.** For example, under the market with SDPs, big CPs have advantages over small CPs. But when we take variety into consideration, the advantages will decrease if users have a greater love for variety.
- **CP sponsored levels are affected by the number of CPs.** It is a new result since previous studies have no result on the relationship between the number of CPs and the CPs' choices on the sponsored level. It is important to know this result for a CP who wants to enter a certain market under SDPs.
- **Attention should be paid to different categorization of variety.** A market with a love for variety can be further categorised into variety-lovers, variety-avoiders or variety-free. They have individual results. For example, in certain variety-avoider market, the CP competition decreases, making CPs with higher cost easier to survive.
- **The impact of ISP decisions on CP competition has some new conclusions.** For example, when users are variety-avoiders, a greater data cap by the ISP may lead to an increase in the sponsored level, i.e., the competition among CPs becomes tougher. Intuitively, this is because in a variety-avoider market, the increased amount of traffic does not lead to a matched increase in variety. Thus, the competition increases.

2 RELATED STUDIES

Nowadays ISPs typically obtain the majority of their revenue from users. However, this one-sided pricing model becomes unviable since users have limits on how much they are willing to pay while their demand for bandwidth keeps increasing. The newly proposed SDP is a two-sided model that makes CPs transfer part of their revenue to users so as to revamp the constrained traffic usage [1]. The proposal of SDP and its emerging in practice made it important to study how SDP will affect the market. Early works study the broad impact of SDP on the users, CPs and ISPs. It is shown that SDPs create a more balanced overall market and can vitalize network expansion [12, 21]. Njoroge *et al.* find that through CP-side pricing, ISPs could secure higher surplus and maintain higher investment levels [14]. Hande *et al.* find that subsidizing the user's connectivity costs by pricing CPs benefits both users and CPs [6], since CPs can gain more revenue, e.g., from advertising, when users consume more contents.

Recent studies emphasizes on the impact of SDPs on the competition among different CPs [9, 12, 21, 22]. A study [21] on the competition between one big CP and one small CP observes that SDPs favor big CPs in certain situations. A two-class service model with consideration of Quality of Services (QoS) [22] shows that SDPs may increase the unbalance in revenue distribution between CPs. A model on regulated sponsorship competition among CPs [12] finds that the main reason that certain CPs might be harmed is the high access prices. Joe-Wong *et al.* [9] find that sponsorship favors less cost constrained CPs and more cost constrained users. We find that all these models are limited in the sense that they only consider the price as the factor affecting the decisions of users. This may not be precise. In this paper, we model variety as an intrinsic factor and reconsider the competition among CPs in SDPs. Our new model rectifies some conclusions of the past studies and derives some completely new results.

3 THE GENERAL MODEL

There are three parties in the market of wireless data networks: a set of potential CPs $\mathcal{N}$, where $N = |\mathcal{N}|$ and a specific CP with index i is denoted by CP_i, a set of users $\mathcal{L}$, where $L = |\mathcal{L}|$, and a monopolistic ISP which provides the link capacity μ. We also denote $\tilde{\mathcal{N}}$ as the set of incumbent CPs, where $\tilde{\mathcal{N}} \subseteq \mathcal{N}$ and $\tilde{N} = |\tilde{\mathcal{N}}|$. Then we can denote the system as a quadruple $(\mathcal{N}, \tilde{\mathcal{N}}, \mu, L)$. In this paper, we care about variety demand, rather than competition between ISPs, so we focus on the situation with a single ISP, like [5, 22].

In this section, we first model the behaviors of end users. And we introduce how to use RLV to capture their variety demand. We then model the utility and behaviors of the CPs and the ISP. Finally, we model the overall market as a two-stage Stackelberg game.

3.1 The Behaviors of End Users

We use a *time* vector $\mathbf{t} = t_{i \in \mathcal{N}}$ to represent a user's consumption in contents of different CPs. Here t_i indicates user's access time in a certain CP_i during a fixed period, e.g., one month. We note that rather than the traffic volume, the utility of end users depends on

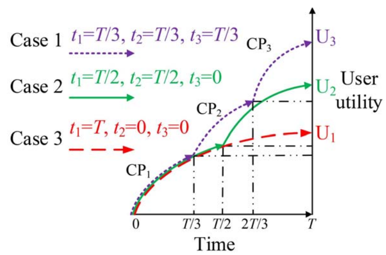


Figure 2: An illustration for RLV. Given the same amount of total time T, user utility will increase if he/she consumes more different contents, since the marginal utility u' always decreases with more time consumed.

the access time, i.e., t_i. We thus define the user's utility function as $u(t_i)$. We assume that $u(0) = 0$ and $u(t_i)$ is a strictly increasing and concave function. Intuitively, a longer time means a higher user utility, but a smaller marginal user utility. We assume that the utilities from different CPs are additive. Then the aggregated utility of a user is $\sum_{i \in \mathcal{N}} u(t_i)$.

3.1.1 Variety/RLV. We now introduce the *variety* used in this paper. Intrinsically, One challenge is to quantify the willingness of a user to exchange one content x to another content y. We address the exchange of two contents by using the concept of *elasticity*. Another challenge is that each user consumes different contents and we need to quantify the willingness of exchanging multiple contents. We address this issue by using a benchmark: each content is first exchanged to this benchmark. In this paper, we select *time* as the benchmark. This naturally follows the practice as well, since time can be used to view any contents. We now present the formal definitions.

Definition 3.1 (Elasticity). For two variables x and y, the x-elasticity of y is define as $\epsilon_x^y = -\frac{\partial y}{\partial x}\frac{x}{y}$.

The elasticity can be interpreted as the percentage change in y in response to the percentage change in x. The larger elasticity implies y is more sensitive to variation of x. To depict user's preferences for a variety of contents, we define RLV through elasticity.

Definition 3.2 (Relative Love for Variety (RLV)). The user's RLV is the elasticity of the marginal utility with respect to the consumption level t_i,

$$r_u(t_i) = \epsilon_{t_i}^{u'} > 0. \tag{1}$$

As said, we use time as a media to make different contents exchangeable. And we show an illustration to understand RLV in Figure 2. From Definition 3.2, we can see that the value of RLV implies whether users are willing to exchange their access time for higher marginal utility. This follows our inference that the request for a variety of contents is intrinsic to users. A greater value of RLV implies that the user prefers more varieties of contents. The fact of more CPs existing in current market also reflects the user's variety preference. This is an intrinsic characteristic of this market.

3.1.2 Behavior of Users. We define the *rate of traffic consumption* as user's average traffic consumption per unit time towards CP_i, which is usually less than the bandwidth requirement. Different CPs may have different rates of traffic consumption. For example, a user may watch movies on YouTube and do shopping on Amazon for the same time duration, but the traffic he consumed on watching movies is obviously greater than that on shopping. Let α_i be the *rate of traffic consumption* for CP_i. Then, the traffic volume a user consumes on CP_i is $\alpha_i t_i$. With the SDP, the traffic volume can be partially sponsored. Let $h_i \in [0,1]$ be the sponsored traffic fraction provided by CP_i for a user in consuming its content (we call it *sponsored level* hereafter). Let $\bar{h}_i = 1 - h_i$ be user's afforded traffic fraction in consuming the content of CP_i (we call it *afforded level* hereafter). Then, the traffic volume that a user needs to pay for is $\sum_{i \in \mathcal{N}} \bar{h}_i \alpha_i t_i$, which will be accumulated in user's cap quota. Under the present tiered pricing scheme provided by the ISP, each user has a total traffic usage limitation (or data cap) by paying a fixed fee, which is denoted by H, e.g., H = 5GB per month. The additional usage beyond the cap will be charged by a much higher price. In reality, users do usually limit their usage below this cap due to the high fee charged for beyond. Thus, it is reasonable to assume that rational user's usage is below the cap. Under the assumption, user's access fee is a constant and does not affect any result. Therefore, we omit user's access fee in his/her utility formula. Each user also has time limitation on different CPs, e.g., 18% of total usage time is spent on music [16]. The time limitation for CP_i is denoted by $\hat{t}_i$. Taking these constraints into account, an user $l \in \mathcal{L}$ can maximize his/her utility as follows,

$$\begin{aligned} \max_{\mathbf{t}} \quad & \mathcal{U}_l = \sum_{i=1}^{N} u(t_i), \\ s.t. \quad & \sum_{i=1}^{N} \bar{h}_i \alpha_i t_i \le H, \quad t_i \in [0, \hat{t}_i]. \end{aligned} \tag{2}$$

The above optimization can be solved by the Lagrange Multiplier with the optimal solution as follows.

Lemma 3.3. *The optimal access time of a user towards the content from* CP_i*, denoted as* t_i^*,

$$t_i^* = \min\{\hat{t}_i, u'^{-1}(\lambda \bar{h}_i \alpha_i)\}, \tag{3}$$

where λ is the Lagrange multiplier associated with the cap constraint.

Proof of Lemma 3.3. The proofs of this paper (i.e., lemma, proposition, and theorem) can be found in [17]. □

We now study the relationship between RLV and the sponsored level. We first give a definition on Sponsoring-Response Elasticity (SRE) and then link RLV and SRE by Lemma 3.5.

Definition 3.4 (Sponsoring-Response Elasticity (SRE)). . The SRE of a user is the elasticity of time t_i with respect to afforded level $\bar{h}_i$ after sponsoring, i.e., $\epsilon_{\bar{h}_i}^{t_i}$.

Lemma 3.5. *SRE is equal to the inverse of RLV, i.e.,*

$$\epsilon_{\bar{h}_i}^{t_i^*} = -\frac{\bar{h}_i}{t_i^*}\frac{\partial t_i^*}{\partial \bar{h}_i} = \frac{1}{r_u(t_i^*)}, \quad t_i^* \in (0, \hat{t}_i). \tag{4}$$

From the proof [17], we see that both RLV and SRE are linked to the user utility function $u'(t_i)$. This lemma is important because RLV, a separately defined index, can now be integrated into the model and optimization through $\bar{h}_i$.

3.2 RLV Classification and Examples

We see that RLV is related to time. If u' is a concave function, this means that the more content consumption (in terms of time), the greater the RLV. This reflects the situation where the users go online ten hours per day will not spend the entire ten hours on reading emails, but will do a large variety of activities. If u' is a convex function, users will have the opposite characteristics (i.e., they will spend all day on a very small number of activities, e.g., playing games). Accordingly, we further classify users into three categories: *variety-lover*, *variety-avoider* and *variety-free*, to denote RLV increases with t_i (i-RLV, i.e., $r_u'(t_i) > 0$), RLV decreases with t_i (d-RLV, i.e., $r_u'(t_i) < 0$), and RLV is constant with t_i (c-RLV, i.e., $r_u'(t_i) = 0$), respectively.

To depict user's preference of different types of RLV, a general utility function can be defined as follows,

$$u(t_i) = \frac{1}{1-\rho}[(a+t_i)^{1-\rho} - a^{1-\rho}] + bt_i, \tag{5}$$

where $a \geq 0$, $b \geq 0$ and $0 < \rho < 1$.

Here we call ρ as the RLV index. With the different value of parameters a and b, the utility function indicates a certain RLV type of users. For example, when $a = 1$, $b = 0$, the corresponding RLV $r_u(t_i) = \frac{\rho}{1+1/t_i}$ increases with t_i (i-RLV). When $a = 0$, $b = 1$, the corresponding RLV $r_u(t_i) = \frac{\rho}{1+t_i^{\rho}}$ decreases with t_i (d-RLV). When $a = 0$ and $b = 0$, the corresponding RLV is a constant ρ (c-RLV).

3.3 The Behaviors of CPs and the ISP

3.3.1 Utility and Behaviors of CPs. Let v_i be CP_i's revenue obtained from per unit content (we call it *per unit revenue* hereafter). CPs may have quite different per unit revenue, such as Google and YouTube. Some CPs may sponsor different traffic volumes for their users so that more users access more contents. As mentioned above, let the *sponsored level* be h_i. h_i differs from different CPs and $h_i = 0$ means that the CP_i does not participate in the sponsored plan. The cost of CP_i consists of three parts: (i) the cost $q \geq 0$ for the connection service of per unit traffic; (ii) the additional cost $p \geq 0$ for the per unit fee an ISP charges the CPs for the sponsored traffic (we call it *sponsored price* hereafter); (iii) the cost of entry to the market s_i. To homogeneous users, the total traffic usage for CP_i is $L\alpha_i t_i$. In fact, our model is also appropriate for heterogeneous users whose traffic usage is different for different CPs. Here we only consider homogeneous users for mathematical simplicity. Let ϕ_i be the utility function of CP_i, then the decision of CP_i is to choose appropriate h_i to maximize ϕ_i, formally,

$$\max_{h_i \in [0,1]} \quad \phi_i = (v_i - ph_i - q)L\alpha_i t_i - s_i. \tag{6}$$

3.3.2 Utility and Behaviors of the ISP. The revenue of the ISP mainly comes from two sources: the unit price charged to CPs for the connection service, i.e., q, and the sponsored price charged to CPs for the sponsored traffic, i.e., p. Note that we treat the connection service price and the sponsored price for different CPs as equal so as to avoid the arguing about network neutrality rules. We omit the price charged to end users because it is only a constant under the cap scheme. Let the traffic volume transmitted between CPs and users be η and $\eta = \sum_{i \in \mathcal{N}} L\alpha_i t_i$. When the traffic demand exceeds the capacity, i.e., $\eta > \mu$, the system falls into congestion which generates operating costs to ISP. We define the congestion cost as a function $c(\eta, \mu)$, which is convex and monotone increasing in η. In practice, the higher congestion implies worse QoS, thus users may decrease their usage or even transfer to other ISPs, which will reduce the ISP's profit [19]. Then the ISP will consider the negative effects brought by congestion when ISP make decisions. Therefore, we adopt the cost function to depict such profit reduction. Let π be the utility function of the ISP, then the decision of ISP is to choose appropriate p, q to maximize π, formally,

$$\max_{\{p,q\}} \quad \pi = \sum_{i=1}^{N} (ph_i + q)L\alpha_i t_i - c(\eta, \mu). \tag{7}$$

One choice of $c(\eta, \mu)$ is the capacity sharing congestion function [11]. Let load rate ω be the ratio of the traffic demand over capacity, i.e., $\omega = \eta/\mu$. A higher load rate means a higher level of network congestion. Then the congestion cost is defined as $c(\eta, \mu) = \chi\omega^{\delta}$, where χ is a congestion level fee to the ISP and $\delta \geq 1$ represents the load sensitivity. Clearly, $c(\eta, \mu)$ is continuous, increasing in η, decreasing in μ and $c(0, \mu) = 0$, $\lim_{\mu \to \infty} c(\eta, \mu) = 0$. We assume $c(\eta, \mu)$ is a twice differentiable and convex function with respect to ω.

3.4 A Two-Stage Stackelberg Game Model of the Market

The wireless data network market in Figure 1 can be modeled as a two-stage Stackelberg game. In the first stage, the monopolistic ISP is the first mover and CPs are the followers. The ISP decides the sponsored price for CPs, and the data cap for end users, i.e., its strategy profile is $\mathcal{S}^I \in \{(p, H)\}$. In the second stage, the CPs form a simultaneous game themselves. Each CP_i decides the sponsored level for end users, i.e., its strategy profile is $\mathcal{S}_i^P \in \{h_i\}$. The outcome is determined by backward induction. In the second stage, $\mathcal{S}^I$ is considered to be fixed. Each CP decides its optimal sponsoring strategy. Then, in the first stage, the ISP decides its optimal price and data cap based on the outcome of the CPs decisions.

Note that we do not include the decision of q into the ISP's strategy profile. This is because we want to focus on the sponsored data scheme provided by CPs, which influences end user's decisions, but has limited impacts on q. Therefore, we assume q is predetermined and known. More precisely, we emphasize on the competition among CPs, i.e., the simultaneous game in the second stage of the game, which is analyzed in Section 4. And we analyze the ISP decisions and the impact of ISP decisions on the competition among CPs in Section 5.

4 COMPETITION AMONG CPS

Unlike previous pricing models which focus on the interactions among ISPs, SDPs introduce CPs into pricing traffic volumes. Therefore, we focus on the CPs behaviors in this section. We first study the market with homogeneous CPs, i.e., the CPs with the same rate of traffic consumption α and the same per unit revenue v. We also assume that they have the same entry cost s. For example, we can consider the CPs that provide video services to be homogeneous since they have the same rate of traffic consumption. Note that these CPs can provide different contents, thus the market has variety. This scenario is useful since CPs with video services are heavily affected by this new SDP pricing model and they are mostly eager to understand the impact of SDPs on their competition. If we consider that only CPs with video services conduct sponsorship, then it is a market with homogeneous CPs. In addition, we study the market with heterogeneous CPs, which is a general and comprehensive case.

For each market, we analyze both the short-run and the long-run equilibrium states. In the short-run equilibrium, the number of CPs is fixed and no CP in the market finds it profitable to change its sponsored level unilaterally. In the long-run equilibrium, CPs can enter and exit freely, till no new CP wants to join or existing CPs want to leave.

We now first analyze the optimal decision of CPs, and then analyze the equilibrium state of the simultaneous game of the CPs. These help our analysis in Subsection 4.1 and Subsection 4.2 on the detailed CP behaviors.

From Equation (3), we have $t_i^* = u'^{-1}(\lambda \bar{h}_i \alpha_i)$ for any $t_i^* \in (0, \hat{t}_i)$. We can see that user's optimal time varies with sponsored level. For the mathematical simplicity, we treat the t_i as t_i^* hereafter. Thus, we have

$$\bar{h}_i = \frac{u'(t_i)}{\lambda \alpha_i}. \quad (8)$$

With Equation (8), the optimization problem of CP_i, i.e., Equation (6), is rewritten by

$$\max_{t_i \in (0, \hat{t}_i)} \phi_i = (\frac{u'(t_i)}{\lambda} - \alpha_i A_i) p L t_i - s_i, \quad (9)$$

where $A_i = \frac{p+q-v_i}{p}$. Here, we abuse the notation a little and let $z_i = \alpha_i A_i$ be the cost of CP_i (we also call it CP's *type*). If $z_i > 0$, CP_i has a positive cost. A higher (lower) cost usually indicates higher (lower) α_i and smaller (higher) v_i, which demonstrates CP_i has a smaller (higher) advantage in the market competition. If $z_i < 0$, CP_i has a negative cost, i.e., it always benefits from more traffic usage.

Note that when making decisions on its optimal sponsored level, a CP may influence the multiplier λ and the traffic consumption of other CPs. Nevertheless, we consider the case where the number of CPs is large and such influence is ignorable. For example, there were about 2.2 million apps available to download in Apple Store and users had an average of 88.7 apps installed on their smartphones [10, 15]. Thus, we assume that CPs are price takers who are not influential enough to affect the market price, like [4, 7, 20]. Under the assumption, each CP accurately treats the multiplier λ as an exogenous parameter and estimates the equilibrium value of λ. Having done this, the CP behaves like a monopolist on its market and thus maximizes its profit.

Let $D_i \equiv \frac{\partial u(t_i)}{\partial t_i}$, $D_i' \equiv \frac{\partial D_i}{\partial t_i}$. The first-order condition of ϕ_i respects to t_i can be written as

$$D_i + t_i D_i' = [1 - r_u(t_i)] D_i = \lambda \alpha_i A_i. \quad (10)$$

Recall that we have assumed that the user utility function is strictly concave, which implies that $D > 0$ and $D' < 0$. It is thus sufficient to assume that the following Inada conditions [8] hold as follows,

$$\lim_{t_i \to 0} D_i = \infty, \quad \lim_{t_i \to \infty} D_i = 0. \quad (11)$$

When $\lambda \alpha_i A_i > 0$, we have

$$0 < r_u(t_i) < 1, \text{ for any } t_i. \quad (12)$$

The conditions (11) and (12) imply that

$$\lim_{t_i \to 0} (1 - r_u(t_i)) D_i = \infty, \quad \lim_{t_i \to \infty} (1 - r_u(t_i)) D_i = 0. \quad (13)$$

The intermediate value theorem implies that Equation (10) has at least one positive solution. When $\lambda \alpha_i A_i < 0$, the optimal time for CP_i approaches the maximum time $\hat{t}_i$. Furthermore, if the user utility function is strictly concave, Equation (10) has a unique solution and this solution makes the CP's profit achieve the maximum value. The uniqueness condition of the solution is equivalent to

$$r_{u'}(t_i) = -t_i \frac{D_i''}{D_i'} < 2. \quad (14)$$

In summary, we have the following lemma.

LEMMA 4.1. *If the conditions (11) (12) and (14) are satisfied, then for any $\lambda > 0$, there exists a unique optimal decision in equilibrium for CP_i in Equation (9), given by*

$$h_i = 1 - \frac{u'(t_i)}{\lambda \alpha_i}, \ t_i = \min\{u'^{-1}(\frac{\lambda \alpha_i A_i}{1 - r_u(t_i)}), \hat{t}_i\}. \quad (15)$$

Lemma 4.1 shows the sufficient conditions for the uniqueness of each CP's optimal decision. In fact, the condition (14) itself can guarantee such uniqueness. The conditions (11) and (12) guarantee that the optimal decision is reasonable and meaningful. For example, if $r_u(t_i) > 1$ for all $t_i \geq 0$, then for any $A_i > 0$ (even for $v_i > q$), $t_i = 0$. In other words, this means the CPs achieve its maximal profit when no user accesses its content. Clearly, this contradicts to the common sense. Note that the optimal decision here may not be in the equilibrium unless λ is the equilibrium value. In the next subsections, we analyze the optimal decisions of CPs in the equilibrium state.

4.1 Homogeneous Content Providers

We now analyze the market with homogeneous CPs which have the same features of α and v. Note that the same α and v do not imply that the CPs provide identical contents.

4.1.1 The Short-run Equilibrium. In the short-run market, the quantity of incumbent CPs is fixed, that is, $\tilde{N}$ is a constant. We first study the optimal decision of the CPs in the equilibrium state. We then analyze the impact of CP quantity on the short-run equilibrium under the variety preference.

We have known Equation (10) has a single solution t_i. Note that all CPs are homogeneous and face with the same λ, so t_i and h_i are symmetric in equilibrium for all $i \in \tilde{\mathcal{N}}$. Let t and $\bar{h} = 1 - h$ be the symmetric results for end users and CPs. In the equilibrium, if the time maximum is not reached, the cap should be fully filled, i.e.,

$$t = \frac{H}{\tilde{N} \alpha \bar{h}}. \quad (16)$$

With this condition and Lemma 4.1, we can estimate the λ in the equilibrium and thus the optimal solution. More specifically, we have the following proposition.

PROPOSITION 4.2. *In the market with homogeneous CPs, the optimal solution in the equilibrium is*

$$\bar{h} = \max\left\{\frac{A}{1 - r_u(\frac{H}{\tilde{N}\alpha\bar{h}})}, \frac{H}{\tilde{N}\alpha\hat{t}}\right\}, \tag{17}$$

where $t = \min\left\{\frac{H}{\tilde{N}\alpha\bar{h}}, \hat{t}\right\}$.

This proposition captures the characteristic of CP's optimal solution in the equilibrium. If $t < \hat{t}$, the RLV affects the optimal sponsored level. This condition is satisfied if and only if $\bar{h} > \frac{H}{\tilde{N}\alpha\hat{t}}$. If $t = \hat{t}$, the sponsored level also approaches its maximum, i.e., $1 - \frac{H}{\tilde{N}\alpha\hat{t}}$.

THEOREM 4.3 (CP QUANTITY EFFECT). *In the short-run equilibrium, if $t < \hat{t}$, then the sponsored level is higher (lower) in the market with the larger quantity of incumbent CPs when $r'_u > 0$ ($r'_u < 0$). Otherwise, the sponsored level is always proportional to the quantity of incumbent CPs.*

When users are variety-lovers, i.e., $r'_u > 0$, the larger $\tilde{N}$ in the market means smaller consumption level and thus smaller RLV. The variety of contents are better substituted with each other and the competition is more intense. Under this circumstance, the CPs have to increase the sponsored level. On the contrary, when users are variety-avoiders, i.e., $r'_u < 0$, the larger $\tilde{N}$ makes higher RLV. The contents become more differentiated. This time, the competition is mild, which makes CPs decrease the sponsored level.

4.1.2 The Long-run Equilibrium. In the long-run market, CPs could enter or exit according to their operating profit, that is, $\tilde{N}$ could change. We first analyze the quantity of CPs in the equilibrium state. Then we study the comparison of the market with and without SDPs in the consideration with RLV.

When a potential CP can earn positive profit, it will enter the market, which reduces the revenue of incumbent CPs. In the equilibrium, no CP has the incentive to enter the market, i.e., all CPs in the market earn zero profit. More formally,

$$(\bar{h} - A)pL\alpha t = s. \tag{18}$$

With Equation (18) and Proposition 4.2, we can capture the equilibrium number of CPs in the long-run market by the following proposition.

PROPOSITION 4.4. *In the market with homogeneous CPs, the number of CPs in the long-run equilibrium satisfies*

$$\tilde{N}^* = \min\left\{\frac{pHLM}{s}, \frac{H}{\alpha\hat{t}A + \bar{s}/p}\right\}, \tag{19}$$

where $M = r_u\left[\frac{s}{L\alpha p}\frac{1}{A}\left(\frac{1}{M} - 1\right)\right]$.

There are two cases in the equilibrium. When $t < \hat{t}$, the RLV affects the number of CPs in the equilibrium. In particular, if the RLV is a constant, then the number of CPs is independent with the characteristics of CPs. When $t = \hat{t}$, if some new CP enters the market, the sponsored level will become higher according to Theorem 4.3. This reduces all CPs' revenue. And the negative profit prevents this new CP entering the market.

Next, we assume that CPs do not participate in the SDP and the equilibrium quantity of these CPs is denoted as $\tilde{N}^{no}$, then we have the following theorem.

THEOREM 4.5 (MARKET VARIETY). *In the long-run market, if $A \geq 0$, then $\frac{\tilde{N}^{no}}{\tilde{N}^*} > 1$ and decreases with RLV; otherwise, the relationship is reversed.*

From literatures [21, 22], we know that the SDPs have the positive effect of attracting users. This is true when the market consists of the negative-cost CPs, which have high per unit revenue and low rate of traffic consumption, like Google search. However, when the market consists of positive-cost CPs, the SDPs enforce the competition and increase the operating cost to CPs simultaneously. Finally, more CPs exit the market. Nevertheless, when users prefer a greater RLV, the gap between the equilibrium number of CPs in the market with and without SDPs is reduced.

4.2 Heterogeneous Content Providers

We now study heterogeneous CPs which differ in α_i and v_i. CP_i with a larger v_i has potential to sponsor more so as to obtain more competitive advantages. Clearly, α_i depends on the type of contents, e.g., video as compared to email. α_i can also be considered as an indicator of the technology of a CP, especially for the same type of contents. Considering two video CPs with same per unit revenue provide the same content for users. A smaller α_i may mean that CP_i has advanced video coding technology of transmitting the same video in a smaller traffic volume, thus CP_i has more competitive advantages. And we will study how these factors affect the competition.

4.2.1 The Short-run Equilibrium. We start from the short-run scenario and study how SDP and RLV affect the competition among heterogeneous CPs. We first derive the market equilibrium. When the set of CPs $\tilde{\mathcal{N}}$ is given, the market equilibrium should satisfy the following conditions: (i) each user maximizes his/her utility subject to the data cap constraint; (ii) no CP can increase its profit by unilaterally changing its sponsored level.

LEMMA 4.6. *If the conditions (11) (12) and (14) are satisfied, there exists a unique λ such that the market is in the equilibrium.*

This lemma guarantees the uniqueness of equilibrium λ in the short-run market. Combined with Lemma 4.1, each CP's optimal sponsored level can be uniquely determined. We now study how heterogeneous CPs differ their strategies under the optimal decisions by the following theorems.

THEOREM 4.7 (DIFFERENTIATED SUBSIDY). *In the short-run market, for any CP_i and CP_i, where $i, j \in \tilde{\mathcal{N}}$, the sponsored level in the equilibrium satisfies*

i) *If $\alpha_i = \alpha_j$ and $v_i > v_j$, then $h_i > h_j$;*
ii) *If $\alpha_i < \alpha_j$ and $v_i = v_j$, then $h_i > h_j$ when $r'_u < 0$ and $h_i < h_j$ when $r'_u > 0$.*

In this theorem, $\alpha_i = \alpha_j$ and $v_i > v_j$ indicates that CP_i and CP_j are of similar type of contents, e.g., all videos, yet CP_i has a greater per unit revenue as compared to CP_j. In such situation, CP_i will sponsor more. Also in this theorem, $\alpha_i < \alpha_j$ and $v_i = v_j$ indicates that the CP_i has better technology level and the same per unit revenue. In such situation, the sponsored level is dependent with the market types, that is, $h_i > h_j$ in the variety-avoider market and $h_i < h_j$ in the variety-lover market.

Next we study the competition of CPs with and without SDPs. Let ϕ_i^{no} be the utility of CP_i where the market has not adopted the SDPs.

Theorem 4.8 (Market Fairness). *In the short-run market with constant RLV ρ (or ρ'), for any CP_i or CP_j, where $i, j \in \tilde{\mathcal{N}}$ such that $0 < \alpha_i A_i < \alpha_j A_j$, we have the following results in the equilibrium,*

i) $\frac{\phi_i}{\phi_j} = \frac{\phi_i^{no}}{\phi_j^{no}}$ *if* $v_i = v_j$, *and* $\frac{\phi_i}{\phi_j} > \frac{\phi_i^{no}}{\phi_j^{no}}$ *if* $\alpha_i = \alpha_j$;

ii) *For any* $\rho < \rho'$, *we have* $\frac{\phi_i}{\phi_j} > \frac{\phi_i'}{\phi_j'} > 1$.

This theorem considers CP_i and CP_j in the market with constant RLV, where the content of CP_i incurs a smaller cost as compared to CP_j. The first part of this theorem shows when the two CPs have the same profitability, SDPs will not increase the differences of their revenue. It also states that a CP with a higher revenue always has a larger difference via SDPs. In other words, the market becomes more unfair. The second part of this theorem shows that the advantage of big CP_i under SDPs is reduced as users prefer larger RLV (the gap of CP_i and CP_j becomes smaller when ρ increases).

4.2.2 The long-run equilibrium. In this section, we study the impact of user quantity on the long-run equilibrium under the variety preference. For the CPs with the same type. Let z and $\Gamma(z)$ be the random variable of types and the distribution of z over $\mathcal{N}$, respectively. To derive the equilibrium, we assume that there exists a cutoff cost $\bar{z}$ such that for any CP_i, if $z_i < \bar{z}$, CP_i will stay in the market; otherwise, it will leave the market. Then, we have following lemma.

Lemma 4.9. *If conditions (11) (12) and (14) are satisfied, there exists a unique pair $(\bar{z}, N)$ in the market equilibrium.*

This lemma guarantees the uniqueness of the market equilibrium. Then, we can analyze the impact of user quantity on the long-run equilibrium.

Theorem 4.10 (User Quantity Effect). *In the long-run market, if $r' > 0$ ($r' < 0$), then the cutoff cost decreases (increases) with L and the sponsored level increases (decreases) with L.*

As the user quantity is large, each CP in the market earns profit, which attracts more CPs entering the market. This makes smaller traffic consumption for each CP. When the users are variety-lovers, the RLV becomes smaller, which makes the competition tougher. The CPs need to sponsor more so as to survive in the market, which triggers the exiting of the higher-cost CPs. On the contrary, when the users are variety-avoiders, a large market softens competition, which allows higher-cost CPs to enter.

In summary: We once again prove some existing results of previous studies when we take variety into consideration. For example, under the influence of SDPs, the number of CPs in the market decreases if each CP has positive cost (Theorem 4.5) and big CPs (i.e., CPs with higher per unit revenue) have advantage over small CPs (Theorem 4.7). But, the influence of SDPs will be reduced if users have a greater RLV (Theorem 4.5, Theorem 4.8). Furthermore, under SDPs, the advantages of CPs with better technology may decrease when users have higher variety demand (Theorem 4.8), which shows that the excessive RLV will cut down the benefit of technology as well. In addition, we get some new results. After taking variety into consideration, we find that the CP quantity affects the sponsored level (Theorem 4.3). In previous studies, it is previously believed that the CP quantity is independent from the sponsored level. And we find that both the sponsored level and the number of CPs are affected by the user quantity (Theorem 4.10). And details can be found in each theorem.

5 THE IMPACT OF ISP'S STRATEGY

In this section, we study the monopolistic ISP's best strategy and its impact on the market. We first analyze the short-run market where the $(\mathcal{N}, \tilde{\mathcal{N}}, \mu, L)$ keeps unchanged. After that, we analyze the long-run market where the $(\mathcal{N}, \tilde{\mathcal{N}}, \mu, L)$ can be changed. We analyze the market with homogeneous CPs at first and then carry out the evaluation to analyze the market with heterogeneous CPs.

5.1 The Short-run Market

In the short-run market, there exists a fixed number of CPs. Their optimal decisions are significantly affected by the ISP's strategy, and thus affects user's time usage. Based on the best responses of CPs and users, the ISP decides its optimal strategy to maximize its revenue. We first consider the homogeneous market and derive the following theorem.

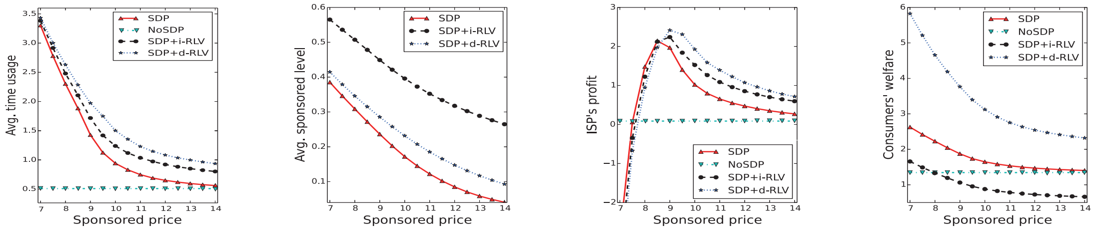
Theorem 5.1 (Short-run Impact). *If $t < \hat{t}$ in the short-run equilibrium, the impact of ISP's strategy satisfies*

i) $\frac{\partial t}{\partial p} < 0$ *and* $\frac{\partial t}{\partial H} > 0$;

ii) $\frac{\partial h}{\partial p} < 0$ *and* $\frac{\partial h}{\partial H} > (<)\ 0$ *when* $r_u' < (>)\ 0$.

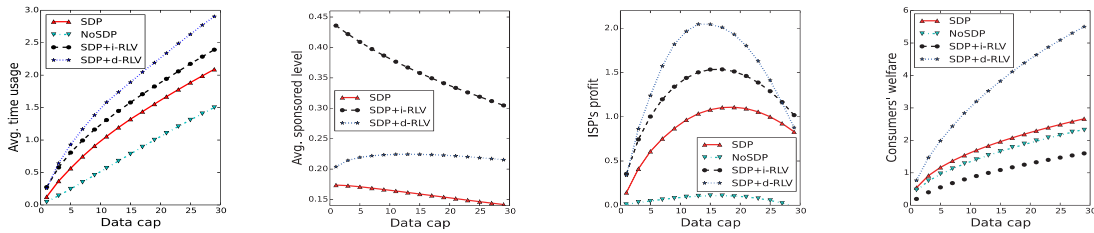
Theorem 5.1 states that both the users' time usage and the CPs' sponsored level decrease with the sponsored price. When the ISP increases the data cap, users' time usage always increases until it reaches the maximum. However, the CPs' sponsored level depends on users' RLV categories. The intuition is that a larger data cap can make users prefer a smaller RLV in the variety-avoider market. The CPs can be substituted more easily and thus the competition becomes more intense. In particular, when the market belongs to the variety-free category, the CPs' sponsored level is independent with ISP's data cap.

We now use simulations to understand the short-run market with heterogeneous CPs. We consider the market with $N = 100$ CPs and one ISP to explore the key features of the market. The per unit revenue of each CP is randomly selected from [\$1, \$10] [1, 9]. The rate of traffic consumption of each CP is randomly selected from [0.05, 0.5] (GB/hour), e.g., watching online movies on smartphone through 4G may consume the volume of 350MB traffic per hour [18]. We adopt user's utility function in Equation (5), with the parameter ρ for each CP randomly distributed over [0.2, 0.8] and $(a, b) = (1, 0)$ for variety-lovers and $(a, b) = (0, 1)$ for variety-avoiders. We set user's maximum consumption time for one CP in the scope of [$1h$, $20h$] [3]. CP's connection service fee and user's data cap are set as \$1/GB and 10GB [2], respectively. We adopt the capacity sharing congestion function and let the congestion level fee be



(a) Time usage vs. p (b) Sponsored level vs. p (c) ISP's profit vs. p (d) Consumer welfare vs. p

Figure 3: Impact of p on the short-run market.

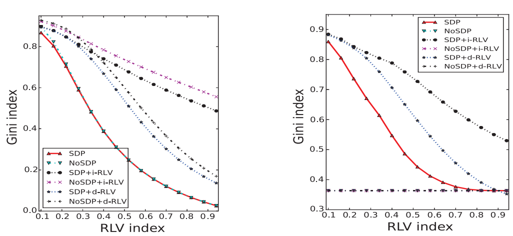


(a) Time usage vs. H (b) Sponsored level vs. H (c) ISP's profit vs. H (d) Consumer welfare vs. H

Figure 4: Impact of H on the short-run market

$\chi = 10$ and the load sensitivity be $\delta = 3$. Note that our simulations do not depend on particular settings, and our purpose is to show qualitative trends in general.

5.1.1 The Impact of ISP's Strategy (p, H) on the Short-run Market. Figure 3 illustrates the impact of sponsored price on the short-run market. As shown in Fig 3(a) and Figure 3(b), the average time usage and the sponsored level always decrease with p increasing. And Figure 3(c) shows that suitable p is required for ISP, e.g., $p = 8.5$ can maximize ISP's profit under the variety-free markets. Figure 3(d) shows that the consumers' welfare always decreases with p increasing since the higher sponsored price limits user's traffic usage.



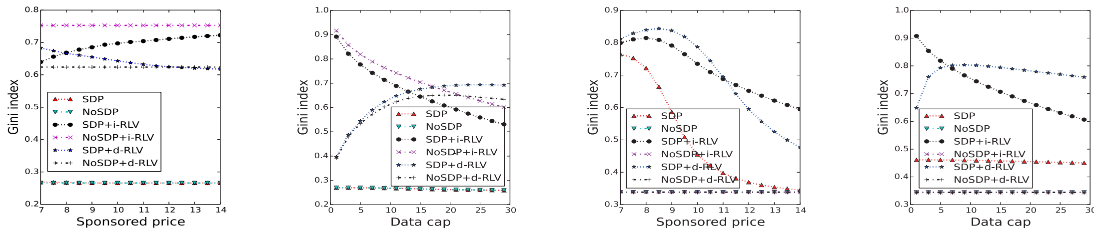
(a) $\alpha_i \neq \alpha_j$ and $v_i = v_j$ (b) $\alpha_i = \alpha_j$ and $v_i \neq v_j$

Figure 5: Impact of ρ on the Gini index in short-run market.

From Figure 4(a), we can see when the ISP increases H, users' time usage always increases. And different lines show that the SDP increases user's time usage further. Figure 4(b) shows that the sponsored level always has a decreasing trend even for variety-free markets, which may be contrary to Theorem 5.1. The intuitive behind is that the maximum time usage for some CPs are approached. Higher data cap usually means the competition among CPs becomes milder. Users can approach the maximum time usage by sponsored less. Even under the time usage constraint, the sponsored level still increases with data cap under the variety-avoider market, especially when data cap is small. Figure 4(c) shows that suitable H for ISP is different under different market. Figure 4(d) shows that users' welfare always increases with H increasing. In addition, both Figure 4(c) and Figure 4(d) show that the ISP and users always benefit from SDPs, especially when users are variety-avoiders.

5.1.2 Market Fairness. We adopt widely known metric, Gini index [13], to measure the fairness. Higher Gini index indicates smaller fairness. The Gini index equals to 0 implying extreme fairness while the Gini index equals to 1 implying extreme unfairness. Figure 5 shows the impact of variety indicator ρ on the market fairness. Both Figure 5(a) and Figure 5(b) show that users woluld prefer a larger RLV, because the market becomes more fair. Figure 5(a) shows that when CPs only have different rates of traffic consumption, i.e., $\alpha_i \neq \alpha_j$, the fairness gap between the market with SDPs and the market without SDPs may keep the same, or becomes larger. Figure 5(b) shows that when CPs only have different per unit revenues, i.e., $v_i \neq v_j$, the fairness of the market with SDPs approaches that of the market without SDPs.

Figure 6(a) and Figure 6(b) show the fairness, where CPs only have different rates of traffic consumption, i.e., $\alpha_i \neq \alpha_j$, which can be considered as an indicator of the technology of a CP. Generally, it is better for the market to encourage the unfairness caused by technical difference, because an unfair market can encourage CPs to improve the technology and reduce the bandwidth required. Figure 6(a) shows that SDP doesn't always increase the unfairness in the market, which only happens when users are variety-avoiders



(a) $\alpha_i \neq \alpha_j$ and $v_i = v_j$ (b) $\alpha_i \neq \alpha_j$ and $v_i = v_j$ (c) $\alpha_i = \alpha_j$ and $v_i \neq v_j$ (d) $\alpha_i = \alpha_j$ and $v_i \neq v_j$

Figure 6: Impact of (p, H) on the Gini index in short-run market.

(SDP+d-RLV line is above NoSDP+d-RLV). When users are variety-lovers, SDP makes the market more fair. It also shows that higher sponsored price makes smaller difference of fairness between the market with SDP and that without SDP. Figure 6(b) illustrates the market becomes more unfair (fair) with increasing of H when users are variety-avoiders (variety-lovers). In addition, the fairness gap between the market with and without SDP becomes larger. Figure 6(c) and Figure 6(d) show the fairness, where CPs only have different per unit revenue, i.e., $v_i \neq v_j$. Figure 6(c) illustrates that SDP always makes the market more unfair. This may result in an unhealthy market since the market prefers the rich CPs if the SDPs are adopted. Fortunately, the unfairness can be alleviated when the sponsored price is higher. In addition, when users are variety-lovers, the market also becomes more fair as the ISP enlarges its data cap, as shown in Figure 6(d). However, when users are variety-avoiders, the unfairness may increase.

5.2 The Long-run Market

In the long-run market, the incumbent number of CPs $\tilde{N}$ is variable. The ISP's strategy can affect the revenue of CPs, which finally affects the equilibrium number of CPs in the market. The ISP can improve its capacity so as to reduce the congestion cost via building more base stations, deploying advanced technology. When we consider the market with homogeneous CPs, the impact of such capacity extension can be obtained by the following theorem.

Theorem 5.2 (Long-run impact). *In the long-run market, we consider the equilibria in $(\mathcal{N}, \tilde{N}, \mu, L)$ and $(\mathcal{N}, \tilde{N}', \mu', L)$ two systems. If $\mu < \mu'$, then we have*

i) *The ISP's strategy satisfies $H \leq H'$ and $p \geq p'$;*
ii) *The number of CPs satisfies $\tilde{N} \leq \tilde{N}'$.*

Theorem 5.2 states that when the ISP expends its capacity, the ISP's optimal p is reduced and H is increased. This will increase the CPs' revenue, thus the market can accommodate more CPs. More CPs in the market will encourage the competition, which leads to higher sponsored level and user's traffic usage. This partially counteracts the effect of capacity expansion. We also use simulations to understand the long-run market with heterogeneous CPs. Most of the basic parameters are the same with the short-run market.

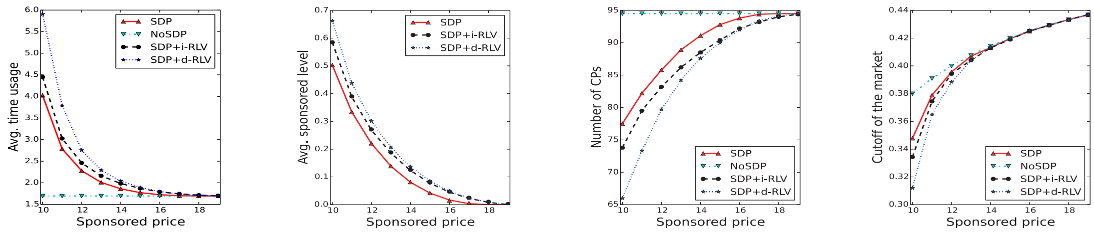
5.2.1 The Impact of ISP's Strategy (p, H) on the Long-run Market. Figure 7(a) and Figure 7(b) show that the average time usage and the sponsored level always decrease with p increasing, which is consistent with the short-run market. And it reduces the burden of each CP due to the sponsored strategy. Thus, each CP's revenue increases, which results in more CPs in the market, as shown in Figure 7(c). Figure 7(d) shows that the cutoff of the market always increases with p increasing, especially under SDP. This indicates that the requirement of the market decreases and more CPs with higher cost can enter the market.

Different from the short-run market, the average time usage and the average sponsored level both increase slightly in the long-run market, as shown in Figure 8(a) and Figure 8(b). The reason behind this is that the number of CPs in the market increases a lot, as shown in Figure 8(c), which counteracts the effects of traffic cap increasing. Due to the operating costs of CPs increasing more slightly, the requirement of entry to the long-run market only has a slight change, as shown in Figure 8(d). It also shows that SDP improves the requirement of entry to the long-run market. In addition, Figure 7(c) and Figure 8(c) also show that the SDP reduces the number of CPs in the market since it improves the requirement of entry to the long-run market.

In summary: The influence of ISP's strategy still cannot be ignored when we consider the variety demand, and we get some new results. In the short-run market, if the ISP increases the sponsored price, both user traffic usage and the sponsored level of CPs decreases. And if the ISP increases the data cap, the user traffic usage increases. These conform to the conclusions of previous studies. But the sponsored level depends on market variety. And for a variety-lover market, a greater data cap leads to a smaller sponsored level. The surprising result is that for a variety-avoider market, a greater data cap may lead to an increase in the sponsored level, i.e., the competition among CPs becomes tougher. Intuitively, this is because the increased traffic usage does not lead to a matched increase in variety. Thus, the competition intensifies. In the long-run market, the number of CPs in the market increases if ISP increases the data cap. The increasing number of CPs in the market counteracts data cap increase. As a result, the sponsored level of CPs and users traffic usage increase slightly as compared with the short-run market.

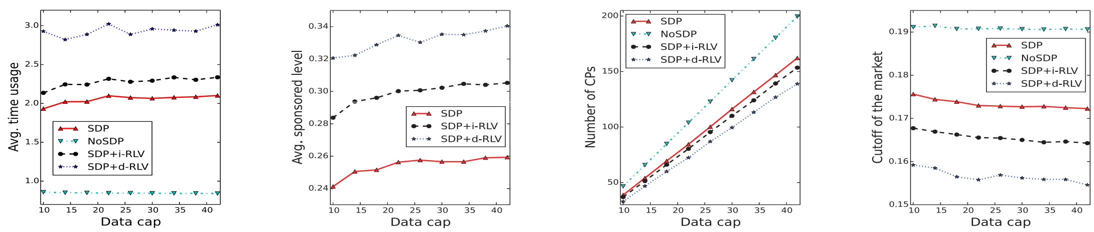
6 CONCLUSION

Previous studies, which are used to understand what SDPs will bring about to the market, only model price as the driving factor for user decision optimization. We argue that they overlook the content variety demand of users. Therefore, we develop a new model to study the competition among CPs under SDPs in this paper. In order to integrate such variety into our new model, we define RLV



(a) Time usage vs. p (b) Sponsored level vs. p (c) Number of CPs vs. p (d) Market cutoff vs. p

Figure 7: Impact of p on the long-run market



(a) Time usage vs. H (b) Sponsored level vs. H (c) Number of CPs vs. H (d) Market cutoff vs. H

Figure 8: Impact of H on the long-run market.

as an index. Through a series of transformation, we integrate RLV into an overall two-stage Stackelberg game model. We conduct a comprehensive analysis on the competition among CPs, then derive a set of results out of our new model, some of which are consistent with (or rectify) previous studies, and some of which are even completely new results, that is, they haven't appeared in previous studies. Overall, the new model proposed in this paper further understands the impact of SDPs on the practical market, and is conducive to stakeholders making decisions.

ACKNOWLEDGMENTS

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